THE HONG KONG POLYTECHNIC UNIVERSITY
香港理工大學

DEPARTMENT OF APPLIED MATHEMATICS
應 用 數 學 系

AMA Seminar

Applications of Wavelets to Numerical PDEs

By

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Abstract: Wavelets have been applied to numerical PDEs since 1990's and offer the advantage of achieving uniformly bounded condition numbers in various Sobolev spaces. However, the general construction of wavelets in bounded domains remained open until year 2021 and the performance of wavelet methods for elliptic problems is only comparable with finite element methods coupled with the multigrid and multi-level methods. In this talk, we first discuss how to construct wavelets on the interval [0, 1] (with or without homogeneous boundary conditions) and then their tensor prod-ucts offer Riesz bases in the Sobolev space $H^1(\Omega)$ with $\Omega = (0, 1)^d$. Next we apply wavelets to the large cavity problem modeled by Helmholtz equation. The solution u to an elliptic interface problem is known to have low global regularity and often only belongs to $H^{1+\epsilon}(0, 1)$ with ϵ not larger than 0.5. Avoiding complicated handling of the interface, we apply wavelets to the elliptic interface problems by a simple strategy. In dimension one, we can prove that the wavelet method can achieve convergence rate $O(h^m)$ in $H^1(\Omega)$ for arbitrary m. In dimension $d \geq 2$, the wavelet method can achieve the convergence rate $O(h^{\frac{0.5d}{d-1}})$ in $H^1(\Omega)$ (In 2D, the scheme is order 1 in H^1 and order 2 in L_2). Some initial numerical experiments are provided to illustrate the performance of wavelet methods for both Helmholtz equation and the elliptic interface problem. This talk is built on several ongoing research projects with Dr. M. Michelle at Purdue University.

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ALL ARE WELCOME