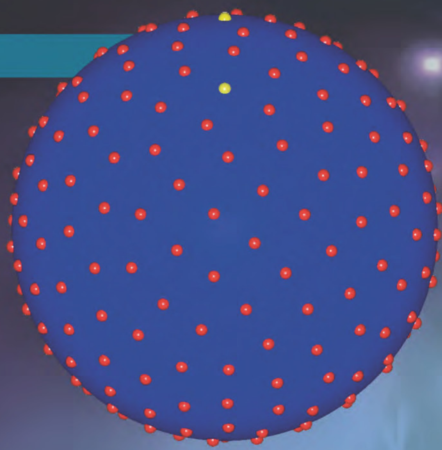
The Hong Kong Polytechnic University
香港理工大學

Department of Applied Mathematics
應 用 數 學 系

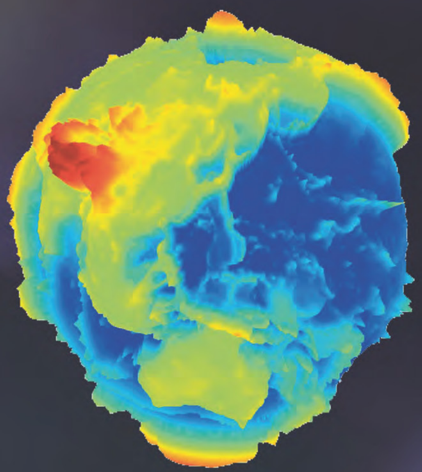
WORKSHOP
on
Approximation and Optimization on the Sphere and Other Surfaces

OBJECTIVES

Approximation and optimization on the sphere and other surfaces involve many challenging problems in mathematics and scientific computation. This workshop is to exchange ideas at the frontier of research on these topics and to enhance research collaboration between various research areas in pure mathematics, computational mathematics and applied mathematics.



SPEAKERS



Raymond Chan	(Chinese University of Hong Kong)
Tony Chan	(Hong Kong University of Science and Technology)
Xiaojun Chen	(The Hong Kong Polytechnic University)
Ka Sing Lau	(Chinese University of Hong Kong)
Franklin T. Luk	(Hong Kong Baptist University)
Michael Ng	(Hong Kong Baptist University)
Liqun Qi	(The Hong Kong Polytechnic University)
Ian Sloan	(The Hong Kong Polytechnic University and The University of New South Wales)
Stephen Smale	(City University of Hong Kong)
Rob Womersley	(The Hong Kong Polytechnic University and The University of New South Wales)

DATE 24 October 2009 (Saturday)

TIME 9:30 – 13:00, 14:30 – 18:00

VENUE Room DE408, The Hong Kong Polytechnic University

SPONSORS
The Hong Kong Polytechnic University, The AMSS-PolyU Joint Research Institute

WORKSHOP WEBSITE http://www.polyu.edu.hk/ama/jri/events.htm

ENQUIRY Ms. Cynthia Hau at 2766 6952 or email at mallhau@inet.polyu.edu.hk

ALL ARE WELCOME

Workshop
Approximation and Optimization on the Sphere and Other Surfaces

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Date: 24 October 2009 (Saturday)
Time: 9:30 -13:00, 14:30 -18:00
Venue: Room DE408, The Hong Kong Polytechnic University

9:30 – 9:35 Opening

(Chair, Liqun Qi)

9:35 – 10:10 **Ian H. Sloan** (The Hong Kong Polytechnic University, The University of New South Wales)
Multiscale Approximation on the Sphere

10:10 – 10:45 **Raymond Chan** (The Chinese University of Hong Kong)
Simultaneously Inpainting in Image and Transformed Domains

10:45 – 11:20 **Michael Ng** (Hong Kong Baptist University)
Numerical Linear Algebra, Data Mining and Image Processing

11:20-11:50 Coffee break

(Chair, Raymond Chan)

11:50 – 12:25 **Franklin T. Luk** (Hong Kong Baptist University)
A Pivoted LLL Algorithm

12:25 – 13:00 **Rob Womersley** (The Hong Kong Polytechnic University, The University of New South Wales)
Spherical L-designs with Roughly L^2/2 Points and High Precision

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13:00 – 14:30 Lunch

(Chair, Franklin T. Luk)

14:30 – 15:05 **Stephen Smale** (City University of Hong Kong)
Optimizing a Balance Cut

15:05 – 15:40 **Ka Sing Lau** (The Chinese University of Hong Kong)
The Laplacian and Heat Kernel on Fractal Set

15:40 – 16:10 Coffee break

(Chair, Ian H. Sloan)

16:10 – 16:45 **Tony Chan** (Hong Kong University of Science and Technology)
Brain Mapping with Conformal and Quasi-conformal Geometry

16:45 – 17:20 **Liqun Qi** (The Hong Kong Polytechnic University)
Minimizing a Multi-Variate Homogeneous Polynomial over the Sphere

17:20 – 17:55 **Xiaojun Chen** (The Hong Kong Polytechnic University)
Computational Existence Proofs for Spherical t-Designs

17:55 – 18:00 Closing
18:00 – 20:00 Dinner

Simultaneously Inpainting in Image and Transformed Domains

Raymond Chan
Department of Mathematics
The Chinese University of Hong Kong
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(Joint work with: Jian-Feng Cai, Lixin Shen, and Zuowei Shen)

Abstract. Inpainting in the image domain refers to the problem of filling in missing or corrupted pixels in an image. It can be regarded as an interpolation problem where we approximate the pixel values of the missing or corrupted pixels by the pixel values of the given uncorrupted pixels. In this talk, we discuss the restoration of images that have missing or corrupted data in either the image domain or the transformed domain or in both. The transform used can be any orthonormal or tight frame transforms. We present an iterative algorithm that can restore the incomplete data in both domains simultaneously. We outline the convergence of the algorithm and its optimal properties. Applications of the algorithm to super-resolution image reconstruction with different zooms are presented.

Brain Mapping with Conformal and Quasi-conformal Geometry

Tony Chan
Hong Kong University of Science and Technology
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(joint work with Ronald Lui and Yalin Wang)

Abstract. In Human Brain Mapping, neuroscientists commonly aim to identify structural differences between healthy and unhealthy brains. Surface-based cortical analysis has been commonly used. In this talk, I will describe how conformal and quasiconformal geometry can be used to analyze brain surfaces effectively. I will firstly describe how conformal and quasiconformal structures can be computed. With harmonic energy minimization, holomorphic 1-form and discrete curvature flow methods, we can parameterize brain surfaces onto various canonical domains such as sphere, Euclidean plane, and Poincare disk. It can be used for various applications such as grid generation, surface registration, feature detection, computation on surfaces, etc.Besides, different surfaces with landmarks correspondence have different conformal structures. This gives ”signatures” for different brain surfaces for shape analysis. In the second part of my talk, I will describe how conformal structure of the brain surface can be used for shape analysis, using Teichmuller theory as a tool. Specifically, I will describe how different shape indices such as Teichmuller shape space coordinates, Beltrami coefficients and multivariate tensor-based morphometry can be computed and used for shape analysis.

Computational Existence Proofs for Spherical t-Designs

Xiaojun Chen
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(This is joint work with Andreas Frommer and Bruno Lang of University of Wuppertal, Germany.)

Abstract. Spherical t-designs provide quadrature rules for the sphere which are exact for polynomials up to degree t. In this paper, we propose a computational algorithm based on interval arithmetic which, for given t, upon successful completion will have proved the existence of a t-design with $(t+1)^2$ nodes and will have computed narrow interval enclosures which are known to contain these nodes with mathematical certainty. Since there is no theoretical result which proves the existence of a t-design with $(t+1)^2$ nodes for arbitrary t, our method contributes to the theory because it was tested successfully for $t = 1, 2, \ldots, 100$, i.e., for all t considered so far. The t-design is usually not unique; our method aims at finding a well-conditioned one. The method relies on computing an interval enclosure for the zero of a highly nonlinear system of dimension $(t+1)^2$. We therefore develop several special approaches which allow us to use interval arithmetic efficiently in this particular situation. The computations were all done using the MATLAB toolbox INTLAB.

The Laplacian and Heat Kernel on Fractal Set

Ka-Sing Lau
The Chinese University of Hong Kong
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(joint work with Alexander Gregorýan and Jaixin Hu)

Abstract. The Laplacian plays a central role in analysis, differential geometry and pde. Recently there is a surge of interest to study this operator and the diffusion on fractal sets. One of the question is how to define the Laplacian without derivative. In this talk we will give a brief survey on this and to report some of our work in connection with heat kernels that arise.

A Pivoted LLL Algorithm

Franklin Luk
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Hong Kong Baptist University
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(This is joint work with Professor Sanzheng Qiao of McMaster University, Canada.)

Abstract. The LLL algorithm was published in 1982 by Lenstra, Lenstra, and Lovasz. It is a lattice basis reduction method with many important applications.

In the 1990s, the LLL algorithm was proposed as a preconditioner for the universal lattice decoding problem. Very roughly, the solution procedure for this decoding problem consists of two steps. For step one, find a reduced basis. For step two, enumerate all lattice points inside a sphere in order to find the lattice point closest to the sphere's center. The LLL algorithm was found to be an effective technique for step one, so that the radius of the sphere for step two would be small.

In 2008 Luk and Tracy developed a matrix interpretation of the LLL algorithm. Building on their work, we propose to add pivoting to the algorithm. We prove that our new algorithm always terminates, and we construct a class of ill-conditioned reduced matrices to illustrate the advantages of pivoting.

Numerical Linear Algebra, Data Mining and Image Processing

Michael Ng
Department of Mathematics
Hong Kong Baptist University
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Abstract. In this talk, I will present two applications in data mining and image processing. The first application is related to the problem where an instance can be assigned to multiple classes. For example, in text categorization tasks, each document may be assigned to multiple predefined topics, such as sports and entertainment; in automatic image or video annotation tasks, each image or video may belong to several semantic classes, such as urban, building, road, etc; in bioinformatics, each gene may be associated with a set of functional classes. The second application is related to multiple class image segmentation problem. For example, foreground-background segmentation has wide applications in computer vision (scene analysis), computer graphics (image editing) and medical imaging (organ segmentation). Both applications share the concept of semi-supervised learning and involve several numerical linear algebra issues. Finally, I will present some matrix computation methods for solving these numerical issues. Experimental results are also given to demonstrate the applications and computation methods.

Minimizing a Multi-Variate Homogeneous Polynomial over the Sphere

Liqun Qi
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Abstract. The minimization problem of a multi-variate homogeneous polynomial over the sphere has applications in optimal control, signal processing, quantum physics and magnetic resonance imaging. This problem is NP-hard with respect to its dimension. When the dimension is small or all the coefficients are non-negative, better results can be obtained. In this talk, we review complexity, algorithms and applications of this problem.

Multiscale Approximation on the Sphere

Ian H. Sloan
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University of New South Wales, Sydney, Australia
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Abstract. This talk describes recent joint work with Q. Thong Le Gia and Holger Wendland, in which we describe, analyse and illustrate a multiscale method for the unit sphere $\mathbb{S}^2 \subset \mathbb{R}^3$. The multiscale approximation is constructed using scaled versions of a single compactly supported radial basis function (RBF) $\Psi(\mathbf{x}, \mathbf{y}) = \psi(|\mathbf{x} - \mathbf{y}|)$ for $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$. It uses a sequence of decreasing scales $\delta_1, \delta_2, \ldots$, and a sequence of point sets $X_1, X_2, \ldots$, with the mesh norm of X_j proportional to δ_j. The approximation is a linear combination of scaled RBFs $\Psi_\delta(\mathbf{x}, \mathbf{y}) := c_\delta \Psi(\frac{|\mathbf{x}-\mathbf{y}|}{\delta})$ with different scales δ, restricted to the sphere $\mathbb{S}^2$. The jth term is the correction at stage j obtained by interpolating the error at stage $j-1$ using RBFs for the finer scale δ_j and the larger point set X_j. While the idea of a multiscale scheme has appeared previously, for example in papers of Schaback, Narcowich/Schaback/Ward, Floater/Iske, and Hales/Levesley and books and papers of Freeden and colleagues, there seems to be no existing analysis of a multiscale approximation based on scaled versions of a single compactly supported RBF and scattered data, for either spherical or Euclidean regions. In this talk I shall outline the ideas behind our error analysis for the sphere, and illustrate the method with a problem from geophysics, that of approximating height above sea level on Earth.

Optimizing a Balance Cut

Steve Smale
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City University of Hong Kong
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Abstract. Binary clusters play a large role in pattern recognition for example. On a manifold the Cheegar decomposition via the second eigenfunction of the Laplacian is one method for attempting such a decomposition. We will explore general settings and optimization algorithms with their error estimates.

Spherical L-designs with Roughly $L^2/2$ Points and High Precision

Rob Womersley
The University of New South Wales, Australia
E-mail: R.Womersley@unsw.edu.au
(This is joint work with Ian Sloan.)

Abstract. Spherical L-designs are equal weight numerical integration rules for the sphere which are exact for all spherical polynomials of degree up to L. They are known to exist if the number of points N is large enough and that the lower bound on N of roughly $L^2/4$ is not achievable for $L \geq 3$. It still not known that spherical L-designs with $N = O(L^2)$ exists for all degrees L.

This talk looks at a variety of variational characterizations based on functions with strictly positive Legendre coefficients and related systems of nonlinear equations. This provides computed spherical L-designs for L up to 138 and symmetric L designs for L up to 181.