The Hong Kong Polytechnic University
Department of Applied Mathematics

Colloquium

On the conversion between permanents and determinants

by

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Abstract

The talk is based on the works [1, 2, 3, 4].
Two important functions in matrix theory, determinant and permanent, look very similar:

$$\det A = \sum_{\sigma \in S_n} (-1)^\sigma a_{1\sigma(1)} \cdots a_{n\sigma(n)} \quad and \quad \operatorname{per} A = \sum_{\sigma \in S_n} a_{1\sigma(1)} \cdots a_{n\sigma(n)}$$

here $A = (a_{ij}) \in M_n(F)$ is an $n \times n$ matrix and S_n denotes the set of all permutations of the set $\{1, \dots n\}$.
While the computation of the determinant can be done in a polynomial time, it is still an open question, if there exists a polynomial algorithm to compute the permanent. Due to this reason, starting from the work by Pólya, 1913, different approaches to convert the permanent into the determinant were under the intensive investigation. Among our results we prove the following theorem:

Theorem 1. Suppose $n \geq 3$, and let F be a finite field with char $F \neq 2$. Then, no bijective map $T: M_n(F) \to M_n(F)$ satisfies $per\ A = \det T(A)$.

Also we investigate Gibson barriers (the maximal and minimal numbers of non-zero elements) for convertible (0, 1)-matrices for the full matrix algebra, symmetric matrices, symmetric fully indecomposable matrices, and solve several related problems.
Our results are illustrated by the number of examples.

[1] M. Budrevich, A. Guterman: Permanent has less zeros than determinant over finite fields, *American Mathematical Society, Contemporary Mathematics*, **579**, (2012) 33-42.
[2] A. Guterman, G. Dolinar, B. Kuzma, Pólya's convertibility problem for symmetric matrices, *Mathematical Notes*, **92**, no. 5, (2012) 684-698.
[3] G. Dolinar, A. Guterman, B. Kuzma, M. Orel, On the Pólya's permanent problem over finite fields, *European Journal of Combinatorics*, **32** (2011) 116-132.
[4] G. Dolinar, A. Guterman, B. Kuzma, M. Orel, Lower bound for Polya problem on permanent, International Journal of Algebra and Computation, 26, 6, 2016, 1237-1255.

Date : 30 August, 2017 (Wednesday)
Time : 11:00a.m. – 12:00noon
Venue : TU801, The Hong Kong Polytechnic University

***** ALL ARE WELCOME *****