

The Hong Kong Polytechnic University  
Department of Applied Mathematics

Colloquium

Extremal Centralizers and their Applications

by

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**Abstract**

Let  $A \in M_n(\mathbb{F})$ , then its centralizer  $\mathcal{C}(A) = \{X \in M_n(\mathbb{F}) \mid AX = XA\}$  is the set of all matrices commuting with  $A$ . For a set  $S \subseteq M_n(\mathbb{F})$  its centralizer  $\mathcal{C}(S) = \{X \in M_n(\mathbb{F}) \mid AX = XA \text{ for every } A \in S\} = \bigcap_{A \in S} \mathcal{C}(A)$  is the intersection of centralizers of all its elements. Centralizers are important and useful both in fundamental and applied sciences. A non-scalar matrix  $A \in M_n(\mathbb{F})$  is *minimal* if for every  $X \in M_n(\mathbb{F})$  with  $\mathcal{C}(A) \supseteq \mathcal{C}(X)$  it follows that  $\mathcal{C}(A) = \mathcal{C}(X)$ . A non-scalar matrix  $A \in M_n(\mathbb{F})$  is *maximal* if for every non-scalar  $X \in M_n(\mathbb{F})$  with  $\mathcal{C}(A) \subseteq \mathcal{C}(X)$  it follows that  $\mathcal{C}(A) = \mathcal{C}(X)$ . We investigate and characterize minimal and maximal matrices over arbitrary fields. Our results are illustrated by applications to the theory of commuting graphs of matrix rings, to the preserver problems, namely to characterize of commutativity preserving maps on matrices, and to the centralizers of high orders.

The talk is based on our recent joint works with G. Dolinar, B. Kuzma and P. Oblak.

**Date : 5 December, 2018 (Wednesday)**

**Time : 2:00pm – 3:00pm**

**Venue : TU801, The Hong Kong Polytechnic University**

**\* \* \* ALL ARE WELCOME \* \* \***