# Eigenvalues of An Even-Order Real Supersymmetric Tensor

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**Abstract**

In this paper, we define eigenvalues and eigenvectors of an even-order real supersymmetric tensor. We show that they are are roots of a one-dimensional polynomial associated with the hyperdeterminant of that supersymmetric tensor. We call this one-dimensional polynomial the characteristic polynomial of that supersymmetric tensor. We show that the product of all eigenvalues is equal to the value of the hyperdeterminant of that supersymmetric tensor. We may use this property to calculate the hyperdeterminant and construct its formula. Real eigenvalues with real eigenvectors are called H-eigenvalues. H-eigenvalues always exist and the supersymmetric tensor is positive definite if and only if all of its H-eigenvalues are positive. An $m$th order $n$-dimensional supersymmetric tensor where $m$ is even has exactly $n(m-1)^{n-1}$ eigenvalues. They are distributed in $n$ balls in $C^n$. These $n$ balls are with the diagonal elements of the supersymmetric tensor as their centers, and the sums of the absolute values of the off-diagonal elements as their radii. The largest (smallest) H-eigenvalue is always in the rightest (leftest) component of the union of these $n$ balls.

*Key words:* Eigenvalue, Supersymmetric Tensor, Hyperdeterminant.

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