DEPARTMENT OF APPLIED MATHEMATICS
應 用 數 學 系

# The Hong Kong Polytechnic University
# Department of Applied Mathematics

# Colloquium

## Fast computation of orthogonal expansions

**by**


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## Abstract

Many partial differential equations of interest are conveniently stated as a Cauchy problem. This is true in particular with regards to equations of quantum mechanics. There are important advantages in solving them with spectral methods, yet such methods are predicated on the availability of fast (i.e. O(n log n) ) expansions. The most enticing possibility is to use a basis of Hermite functions, because they lead to a tridiagonal, skew-symmetric differentiation matrix, thereby ensuring numerical stability, ensuring conservation of energy and allowing for fast computation of time steps.

The challenge is this to expand fast in Hermite functions and this is the theme of this talk. Assuming sufficiently rapid decay of a sufficiently `nice' function at infinity, we first prove the Sombrero Theorem: essentially, even though the function is not periodic, the Fourier coefficients of a sufficiently stretched function decay at a spectral speed until they are very small (typically, much smaller than required accuracy), whereby they assume the expected, exceedingly slow decay. (In logarithmic scale we obtain an outline of a sombrero, hence the name.) Of course, these coefficients can be computed by FFT in O(n log n) operations. We subsequently use connection coefficients to compute the coefficients in a Hermite function expansion in additional O(n) operations.

This is joint work with Helge Dietert.


**Date : 18 November, 2016 (Friday)**
**Time : 11:00a.m. – 12:00noon**
**Venue : TU801, The Hong Kong Polytechnic University**

*** * * ALL ARE WELCOME * * ***