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H_∞ control of T–S fuzzy fish population logistic model with the invasion of alien species

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ABSTRACT

The problem of H_∞ control for a fish population logistic model with the invasion of alien species is studied via a T–S fuzzy control approach in this paper. Firstly, the harvested capability induced by economic factors and purification capability induced by invasion of alien species are analyzed. Secondly, the corresponding bio-economic model is established by taking the above two factors into account. Thirdly, the singularity-induced bifurcation (SIB) and impulsive behavior of the resultant bio-economic model are investigated. After that, a T–S fuzzy system is used to describe the nonlinear system for the bio-economic model with added input disturbance. A sufficient condition is proposed to satisfy H_∞ norm of the system by using the Lyapunov theory and a linear matrix inequality approach. Finally, a modified differential transform method is exploited to present the invasion system to analyze the characters. The application of H_∞ controller according to actual events demonstrates the effectiveness of the method used in this paper.

1. Introduction

During the past decades, the Takagi–Sugeno (T–S) fuzzy systems have attracted considerable attention, see e.g., [1–3]. For the applicability of the T–S fuzzy system theory, [4] surveys the vehicle suspension system problem based on the H_∞ control approach. The authors in [5] further studied the delay-dependent stability criteria and time-varying delay in the T–S fuzzy systems. [6] has addressed the filter design problem for fuzzy systems with D stability constraints. In this paper, we further study fuzzy systems with an H_∞ control approach, which better controls and regulates the densities of species for biological benefit. The problem of how to build an appropriate model, which can give a description reasonably well for the complex situations of interference and uncertainty, is a significant issue for practical control system design. Singular systems play an important role in describing models accurately of the practical applications. The theory of singular systems has been extensively applied to various fields [7–10]. Both the Leontief dynamic input–output model and the Hopfield neural network model are differential algebraic systems (namely singular systems) [11–12]. Moreover, some studies of the various bifurcations in power systems by means of a singular system's approach have produced great significance in revealing the mechanism of voltage instability [13–15]. It is known through [16–18] that the bifurcation theory of singular systems already has had a solid theoretical foundation. Recently, Zhang and other scholars studied several kinds of biological dynamical systems [19–24] by utilizing the theory of singular systems. In general, it is very challenging to combine the knowledge of singular systems and application of H_∞ fuzzy control [25,26] to eliminate the undesirable dynamic behaviors in bio-economic models and analyze the ecological significance, which motivates this paper.

This paper is a study on H_∞control of a T–S fuzzy fish population logistic model with the invasion of alien species. The innovation of this paper is the proposal of the purification capacity for Spartina anglica and the combination of the model and the T–S fuzzy H_∞ control problem. The H_∞control, which we apply to the biological model in this paper, cannot only effectively control the bifurcation but also guarantee the co-existence of fish population and alien species *Spartina anglica* in a certain range. In other words, this control mechanism takes into account the condition that some space should be kept for the survival of *S. anglica*. If the input disturbance and output of the system fluctuate greatly in a short period, then we can adjust the relevant H_∞norm γ to ensure them to survive in a proper range. This reduces the influence of sudden change of species density on the environment to a great extent. The implication of γ for biological significance is the positivity and urgency of government management. At the same time, harvest capacity for fish and purification capacity for *S. anglica* exist for the economic need. *S. anglica*, which is one of the aquatic plants introduced abroad by the

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water conservancy department, is used to promote the sedimentation and reclamation. Now, it has become a big disaster. A few years ago, the shoals of Ling-kun Island were overrun with the *S. anglica* which occupied the tens of thousands of acres of tidal flats. In consequence, fish and shellfish of the tidal flats vanished. It took a lot of manpower and material resources to burn and cut it [27]. In order to protect the ecological environment and guarantee the economic development simultaneously, many biological–economic models have been proposed by some mathematical researchers, and some valuable results have been obtained. For example, the problems of optimal harvesting in the biological economic model are studied in [28–31], which provide the theoretical basis for the reasonable development of biological resources. The existence and stability of equilibrium and positive periodic orbit in the biological economy model are discussed respectively in [32,33]. In reality, the dynamics of systems are often subject to various external disturbances. The existence of external disturbances might deteriorate the system's performance. Thus, many researchers have applied the H_∞ control theory to study these biological–economic models [34]. To investigate the problem step by step, the bio-economic system with added input disturbance has been considered in the later section. The modified differential transform method [35] provides an analytical solution of the invasion model with periodic disturbance in convergent series form. Furthermore, the application of H_∞ controller according to actual events demonstrates the effectiveness of the method used in this paper. Our work is hoped to have a profound impact on better understanding the relationship between alien species and fishery.

This paper is organized as follows. In Section 2, we formulate the fish population logistic model with the invasion of alien species. The proof of singularity-induced bifurcation SIB is carried out in Section 3. The very important steps in our derivation are the fuzzy modeling and the proofs of H_∞ control, which are addressed in Section 4. Section 5 gives a new view to access the invasion model by a modified differential transform method and the illustrations of H_∞ controller for actual events. Finally, we end this paper with some concluding remarks.

2. Modeling

Introducing a density restriction factor $(1-\frac{N}{K})$ to the Malthus model leads to the famous Logistic equation on ecology

$$\frac{dN(t)}{dt}=rN(t)\left(1-\frac{N(t)}{K}\right)$$

where $N(t)$ is the density or quantity of the population at time t, r is the intrinsic growth rate of the population, and K is the environmental capacity. Furthermore, we consider the following logistic model with the invasion of alien species, system (1) below. To be more realistic, we introduce the purification capability for *S. anglica* $E_1(t)$ to system (1):

$$\begin{cases}\frac{dx(t)}{dt}=rx(t)\left(1-\frac{x(t)+y(t)}{K}\right)-E(t)x(t)\\ \frac{dy(t)}{dt}=\beta y(t)-hE_1(t)\\ \frac{dE_1(t)}{dt}=\theta y(t)-\omega E_1(t)-\alpha x(t)\end{cases}\tag{1}$$

where $x(t), y(t), E(t)$ respectively represent the density of fish population, the density of *S. anglica* and the harvested capability to fish at time t. The original water nutrients, water, oxygen and living space are used due to the invasion of alien species, therefore we add $x(t)$ to $y(t)$ for $N(t)$. β represents the intrinsic growth rate of the *S. anglica*. $-hE_1(t)$ represents the quantity of purification to *S. anglica*. $\theta y(t)$ represents the growing degree of *S. anglica*. ω represents the cost of a unit of purification effort. $-\alpha x(t)$ represents that the purification capability is stimulated by the reduction of fish to a certain extent. The constants mentioned above are supposed to be positive.

In 1954, Gordon founded up the open or public fishery economic theory [36]. By this theory, Sustainable Economic Profit = Sustainable Total Revenue − Sustainable Total Cost, when the harvested effort $E(t)$ is given. Therefore, when the harvested effort $E(t)$ switches with time t, we get the following algebraic equation:

$$E(t)(x(t)p-c)=m(t)\tag{2}$$

where p is the market price of the captive population. c is the cost of a unit of capture effort. $m(t)$ is the net economic revenue. Based on (1) and (2), the following singular biological economic system is established:

$$\begin{cases}\dot{x}(t)=rx(t)\left(1-\frac{x(t)+y(t)}{K}\right)-E(t)x(t)\\ \dot{y}(t)=\beta y(t)-hE_1(t)\\ \dot{E}_1(t)=\theta y(t)-\omega E_1(t)-\alpha x(t)\\ 0=E(t)(x(t)p-c)-m(t)\end{cases}\tag{3}$$

Eq. (3) also can be expressed as the following matrix form:

$$A(t)\dot{X}(t)=G(x(t),y(t),E_1(t),E(t)),$$

where

$$X(t)=(x(t),y(t),E_1(t),E(t))^T,$$

$$A(t)=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix},$$

$$G(x(t),y(t),E_1(t),E(t))=\begin{bmatrix}rx(t)\left(1-\frac{x(t)+y(t)}{K}\right)-E(t)x(t)\\ \beta y(t)-hE_1(t)\\ \theta y(t)-\omega E_1(t)-\alpha x(t)\\ E(t)(x(t)p-c)-m(t)\end{bmatrix},$$

The objective of this paper is to analyze the dynamic behavior of the system (3). Based on the understanding the equilibria distribution, the type of bifurcation and other properties, then we further consider the invasion model with added disturbance. H_∞ controller is designed to stabilize bifurcation step by step.

3. The singularity induced bifurcation

For system (3), the phenomenon of economic overfishing [37] appears when $m=0$, which means the revenue equals to the cost. In this case, there exist three equilibria in model (3), which are as follows:

$$P_1(0,0,0,0)$$

$$P_2\left(\frac{K(h\theta-\beta\omega)}{M},\frac{hK\alpha}{M},\frac{K\alpha\beta}{M},0\right)$$

$$P_3\left(\frac{c}{p},\frac{ch\alpha}{p(h\theta-\beta\omega)},\frac{c\alpha\beta}{p(h\theta-\beta\omega)},r-\frac{crM}{Kp(h\theta-\beta\omega)}\right),$$

where

$$M=h\alpha+h\theta-\beta\omega.$$

We pay more attention to local dynamic characteristics near the positive equilibrium of the system in the actual situation of biological economics. The following result has been proven in [38].

Theorem 1. *If* $(h\theta-\beta\omega)>0$, $\left(r-\frac{cr(h\alpha+h\theta-\beta\omega)}{Kp(h\theta-\beta\omega)}\right)>0$, *then* (3) *undergoes the singularity induced bifurcation at* $P_3\left(\frac{c}{p},\frac{ch\alpha}{p(h\theta-\beta\omega)},\frac{c\alpha\beta}{p(h\theta-\beta\omega)},\right.$

$r-\frac{crM}{Kp(h\theta-\beta\omega)}$), and m is a bifurcation parameter so that the system loses stability, where $M=h\alpha+h\theta-\beta\omega$.

Remark: Based on Theorem 1.1 in [39], the other characteristic root of (3) in the neighborhood of $m=0$ is continuous, which means that it will not affect the switch of stability near the equilibrium. And through computation we know that this characteristic root has negative real part. In consequence, the system switches from being stable to unstable as m moves from being negative to positive through zero. For system (3), the Kronecker index of the matrix pencil $\{A(t), J\}$ is 2. The phenomenon of impulse [40] will appear in the system, and at this time the population density will change greatly in an instant, which causes ecological imbalances.

4. H_∞ control

In biology, effective control is to guarantee all the species survival simultaneously. It is consistent with the biological diversity that we control the fish population and *S. anglica* to co-exist at a proper range. This is very significance for the introduction of H_∞ control.

After understanding the equilibria distribution, the type of bifurcation and other properties, now we further consider the invasion model with added disturbance.

Given the following singular biological economic model with the interference factors:

$$\begin{cases}\dot{x}(t)=rx(t)(1-\frac{x(t)+y(t)}{K})-E(t)x(t)+b_{11}\delta(t)\\ \dot{y}(t)=\beta y(t)-hE_1(t)+b_{12}\delta(t)\\ \dot{E}_1(t)=\theta y(t)-\omega E_1(t)-\alpha x(t)+b_{13}\delta(t)\\ 0=E(t)(x(t)p-c)-m(t)+b_{14}\delta(t)\end{cases}\tag{4}$$

where $\delta(t)$ is the input of disturbance, we assume $\delta(t)\in L^2(0,\infty;R)$ (space of square integrable functions) and $b_{11}, b_{12}, b_{13}, b_{14}$ are the disturbed coefficients.

4.1. Fuzzy modeling

In order to facilitate the research, we apply the transformation:

$$\xi_1(t)=x(t)-\frac{c}{p},\quad \xi_2(t)=y(t)-\frac{ch\alpha}{p(h\theta-\beta\omega)},\quad \xi_3(t)=E_1(t)-\frac{c\alpha\beta}{p(h\theta-\beta\omega)},$$

$$\xi_4(t)=E(t)-r+\frac{crM}{Kp(h\theta-\beta\omega)},$$

where

$M=h\alpha+h\theta-\beta\omega.$

Then system (4) transforms into:

$$\begin{cases}\dot{\xi}_1(t)=-\frac{(c+p\xi_1(t))(r\xi_1(t)+r\xi_2(t)+K\xi_4(t))}{Kp}+b_{11}\delta(t)\\ \dot{\xi}_2(t)=\beta\xi_2(t)-h\xi_3(t)+b_{12}\delta(t)\\ \dot{\xi}_3(t)=-\alpha\xi_1(t)+\theta\xi_2(t)-\omega\xi_3(t)+b_{13}\delta(t)\\ 0=p\xi_1(t)(r-\frac{cr(h(\alpha+\theta)-\beta\omega)}{Kp(h\theta-\beta\omega)}+\xi_4(t))-m(t)+b_{14}\delta(t)\end{cases}\tag{5}$$

when $b_{11}=b_{12}=b_{13}=b_{14}=0, m(t)=0$, the equilibrium $P_3\left(\frac{c}{p}, \frac{ch\alpha}{p(h\theta-\beta\omega)}, \frac{c\alpha\beta}{p(h\theta-\beta\omega)}, r-\frac{crM}{Kp(h\theta-\beta\omega)}\right)$ of the system (4) transforms into the origin $(0,0,0,0)$ of system (5). Based on Theorem 1, the phenomenon of singularity induced bifurcation (SIB) will appear in system (5), and $m=0$ is a bifurcation parameter. When m goes from negative to positive, the system will lose stability and the phenomenon of impulse appears.

Applying control to system (5), we get system (6).

$$\begin{cases}\dot{\xi}_1(t)=-\frac{(c+p\xi_1(t))(r\xi_1(t)+r\xi_2(t)+K\xi_4(t))}{Kp}+b_{11}\delta(t)\\ \dot{\xi}_2(t)=\beta\xi_2(t)-h\xi_3(t)+b_{12}\delta(t)\\ \dot{\xi}_3(t)=-\alpha\xi_1(t)+\theta\xi_2(t)-\omega\xi_3(t)+b_{13}\delta(t)\\ 0=p\xi_1(t)(r-\frac{cr(h(\alpha+\theta)-\beta\omega)}{Kp(h\theta-\beta\omega)}+\xi_4(t))-m(t)+b_{14}\delta(t)+u(t)\end{cases}\tag{6}$$

where $u(t)$ is the control variable, which means the management of the government for the development of public resource. The harvested capability properly changes with the application of $u(t)$, thereby purification capacity is affected. In consequence, the density of fish population and *S. anglica* is regulated. Here we only discuss the influence on the disturbance of fish population, in the case of $b_{11}\neq 0,\ b_{12}=0,\ b_{13}=0,\ b_{14}=0$.

Because the singularity induced bifurcation appears in a neighborhood of the equilibrium point, so we suppose that $\xi_1(t)\in[-d,d],\ d>0$. A system of fuzzy equations is given as follows, which can describe system (6) when $\xi_1(t)\in[-d,d]$

Rule 1 If $\xi_1(t)$ is M_1, then $A(t)\dot{\xi}(t)=A_1\xi(t)+B_{11}\delta(t)+B_{21}u(t)$.

Rule 2 If $\xi_1(t)$ is M_2, then $A(t)\dot{\xi}(t)=A_2\xi(t)+B_{12}\delta(t)+B_{22}u(t)$, where $\xi(t)=[\xi_1(t)\quad \xi_2(t)\quad \xi_3(t)\quad \xi_4(t)]^T$, $M_1=\frac{1}{2}(1-\xi_1(t)/d)$,

$$M_2=\frac{1}{2}\left(1+\frac{\xi_1(t)}{d}\right),\quad A(t)=\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix},$$

$$A_1=\begin{bmatrix}-\frac{(c-dp)r}{Kp}&-\frac{(c-dp)r}{Kp}&0&-\frac{c-dp}{p}\\0&\beta&-h&0\\-\alpha&\theta&-\omega&0\\p(r-\frac{cr(h(\alpha+\theta)-\beta\omega)}{Kp(h\theta-\beta\omega)})&0&0&-pd\end{bmatrix};$$

$$A_2=\begin{bmatrix}-\frac{(c+dp)r}{Kp}&-\frac{(c+dp)r}{Kp}&0&-\frac{c+dp}{p}\\0&\beta&-h&0\\-\alpha&\theta&-\omega&0\\p(r-\frac{cr(h(\alpha+\theta)-\beta\omega)}{Kp(h\theta-\beta\omega)})&0&0&pd\end{bmatrix};$$

$$B_{11}=B_{12}=\begin{bmatrix}b_{11}\\0\\0\\0\end{bmatrix};B_{21}=B_{22}=\begin{bmatrix}0\\0\\0\\1\end{bmatrix}.$$

$\lambda_i(\xi_1(t))$ are the membership functions that $\xi_1(t)$ belongs to $M_i (i=1,\ 2)$. In addition, $\lambda_1(\xi_1(t))=\frac{1}{2}\left(1-\frac{\xi_1(t)}{d}\right)$, $\lambda_2(\xi_1(t))=\frac{1}{2}\left(1+\frac{\xi_1(t)}{d}\right)$, $\sum_{i=1}^{2}\lambda_i(\xi_1(t))=1$.

The state equation of the whole system can be expressed as:

$$A(t)\dot{\xi}(t)=\sum_{i=1}^{2}\lambda_i(\xi_1(t))(A_i\xi(t)+B_1\delta(t)+B_2u(t))\tag{7}$$

where $B_1=B_{11}=B_{12},\ B_2=B_{21}=B_{22}$.

4.2. H_∞ control

The following theorem is the main result of this section.

Theorem 2. *For a given $\gamma>0$, if there exist the matrices C, F_i, H_{ij} which satisfy the following linear matrix inequalities:*

$$A^T(t)C=C^TA(t)\geq 0\tag{8}$$

$$C^TA_i^T+F_i^TB_2^T+A_iC+B_2F_i+\frac{1}{\gamma^2}B_1B_1^T<H_{ii}\tag{9}$$

$$A_iC+B_2F_i+B_2F_j+A_jC+\frac{1}{\gamma^2}B_1B_1^T+C^TA_i^T+F_i^TB_2^T+C^TA_j^T$$

$$+F_j^T B_2^T + \frac{1}{\gamma^2} B_1 B_1^T \le H_{ij} + H_{ij}^T \tag{10}$$

$$S_k = \begin{bmatrix} H_{11} & H_{12} & V_{1k}^T \\ H_{21} & H_{22} & V_{2k}^T \\ V_{1k} & V_{2k} & -1 \end{bmatrix} < 0 \tag{11}$$

where C is nonsingular $D = C^{-1}$,H_{ij} is symmetric $H_{ij} = H_{ji}$, $i, j = 1, 2$,$V_{ik} = N_i F_k$, then $u(t) = \sum_{j=1}^{2} \lambda_j(\xi_1(t)) K_j \xi(t)$ makes H_∞ norm of system (12) less than γ. The closed-loop system is quadratically stable when $\delta(t) = 0$. Here $K_j = F_j C^{-1}$, $i, j, k = 1, 2$.

$$\begin{cases} A(t)\dot{\xi}(t) = \sum_{j=1}^{2}\sum_{i=1}^{2} \lambda_i(\xi_1(t))\lambda_j(\xi_1(t))(A_i + B_2 K_j)\xi(t) + \sum_{i=1}^{2} \lambda_i(\xi_1(t)) B_1 \delta(t) \\ z(t) = \sum_{j=1}^{2}\sum_{i=1}^{2} \lambda_i(\xi_1(t))\lambda_j(\xi_1(t)) N_i K_j \xi(t) \end{cases} \tag{12}$$

Proof. firstly, the inequalities (9) and (10) are multiplied by D^T from the left-hand side and multiplied by D from the right-hand side, we can get the following inequalities:

$$A_i^T D + K_i^T B_2^T D + D^T A_i + D^T B_2 K_i + \tfrac{1}{\gamma^2} D^T B_1 B_1^T D < D^T H_{ii} D$$
$$D^T A_i + D^T B_2 K_j + D^T A_j + D^T B_2 K_i + A_i^T D + K_j^T B_2^T D + A_j^T D + K_i^T B_2^T D + \tfrac{2}{\gamma^2} D^T B_1 B_1^T D \le D^T H_{ij} D + D^T H_{ij}^T D.$$

Secondly, based on (11) we get

$$S_k = \begin{bmatrix} H_{11} & H_{12} & \sum_{k=1}^{2} \lambda_k V_{1k}^T \\ H_{21} & H_{22} & \sum_{k=1}^{2} \lambda_k V_{2k}^T \\ \sum_{k=1}^{2} \lambda_k V_{1k}^T & \sum_{k=1}^{2} \lambda_k V_{2k}^T & -1 \end{bmatrix} < 0.$$

Applying the Schur complement Theorem to the above inequality, it can be obtained that

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} < -\begin{bmatrix} \sum_{k=1}^{2} \lambda_k V_{1k}^T \\ \sum_{k=1}^{2} \lambda_k V_{2k}^T \end{bmatrix} \begin{bmatrix} \sum_{k=1}^{2} \lambda_k V_{1k}^T \\ \sum_{k=1}^{2} \lambda_k V_{2k}^T \end{bmatrix}^T \le 0.$$

Multiplying the above formula by $\mathrm{diag}(D^T, D^T)$ from the left-hand side and by $\mathrm{diag}(D, D)$ from the right-hand side, then we get

$$\left[D^T H_{ij} D\right]_{2\times 2} < -\begin{bmatrix} \sum_{k=1}^{2} \lambda_k D^T V_{1k}^T \\ \sum_{k=1}^{2} \lambda_k D^T V_{2k}^T \end{bmatrix} \begin{bmatrix} \sum_{k=1}^{2} \lambda_k D^T V_{1k}^T \\ \sum_{k=1}^{2} \lambda_k D^T V_{2k}^T \end{bmatrix}^T \le 0$$

There exists $u > 0$ so that $\left[D^T H_{ij} D\right]_{2\times 2} < -uI$.

For any $\xi(t) \ne 0, t > 0$, it can be shown that

$$\sum_{j=1}^{2}\sum_{i=1}^{2} \lambda_i(\xi_1(t))\lambda_j(\xi_1(t)) \xi^T(t) D^T H_{ij} D \xi(t) < -z^T(t) z(t)$$
$$\sum_{j=1}^{2}\sum_{i=1}^{2} \lambda_i(\xi_1(t))\lambda_j(\xi_1(t)) \xi^T(t) D^T H_{ij} D \xi(t) < -u\xi^T(t)\xi(t).$$

Construct the singular Lyapunov function $V(t) = \xi^T(t) A^T(t) D \xi(t)$. Based on the inequalities (9) and (10)

$$\begin{aligned} \dot{V}(t) &= (A(t)\dot{\xi}(t))^T D\xi(t) + \xi^T(t) A^T(t) D\dot{\xi}(t) \le \sum_{j=1}^{2}\sum_{i=1}^{2} \lambda_i\lambda_j \xi^T(t)\Big(A_i^T D + K_i^T B_2^T D + D^T A_i \\ &+ D^T B_2 K_i + \frac{1}{\gamma^2} D^T B_1 B_1^T D + D^T A_i + D^T B_2 K_j + D^T A_j + D^T B_2 K_i + A_i^T D + K_j^T B_2^T D \\ &+ A_j^T D + K_i^T B_2^T D + \frac{2}{\gamma^2} D^T B_1 B_1^T D\Big)\xi(t) + \gamma^2\delta^2(t) - \Big(\gamma\delta(t) \\ &- \frac{1}{\gamma}\sum_{i=1}^{2} \lambda_i B_1^T D\xi(t)\Big)^T \Big(\gamma\delta(t) - \frac{1}{\gamma}\sum_{i=1}^{2} \lambda_i B_1^T D\xi(t)\Big) \le \sum_{i=1}^{2}\sum_{i=1}^{2} \lambda_i^2 \xi^T(t) D^T H_{ii} D\xi(t) \\ &+ \sum_{j=1}^{2}\sum_{i=1}^{2} \lambda_i\lambda_j \xi^T(t)(D^T H_{ij} D + D^T H_{ij}^T D)\xi(t) + \gamma^2\delta^2(t) - \Big(\gamma\delta(t) \\ &- \frac{1}{\gamma}\sum_{i=1}^{2} \lambda_i B_1^T D\xi(t)\Big)^T (\gamma\delta(t) - \frac{1}{\gamma}\sum_{i=1}^{2} \lambda_i B_1^T D\xi(t)) \end{aligned} \tag{13}$$

Obviously, when $\delta(t) = 0$, $\dot{V}(t) \le \sum_{j=1}^{2}\sum_{i=1}^{2} \lambda_i\lambda_j \xi^T(t)\ D^T H_{ij} D\xi(t) < -u\xi^T(t)\xi(t)$.

Therefore system (12) is quadratically stable. When $\delta(t) \ne 0$, based on inequality (13) we know that

$$\dot{V}(t) < -z^T(t)z(t) + \gamma^2\delta^2(t) - (\gamma\delta(t) - \frac{1}{\gamma}\sum_{i=1}^{2} \lambda_i B_1^T D\xi(t))^T (\gamma\delta(t) - \frac{1}{\gamma}\sum_{i=1}^{2} \lambda_i B_1^T D\xi(t))..$$

Assume $\xi(0) = 0$, then $V(\xi(0)) = 0$.

Both sides of the above inequalities are integrated from 0 to infinity.

$$0 \le V(t) < -\|z\|_2^2 + \gamma^2\|\delta\|_2^2 - \gamma^2\left\|\delta - \frac{1}{\gamma^2}\sum_{i=1}^{2} \lambda_i B_1^T D\xi(t)\|_2^2.\right.$$

Consequently, $\|z\|_2^2 \le \gamma^2\|\delta\|_2^2$, which means H_∞ norm of system (12) less than γ.

The proof is completed.

5. Application instance

5.1. Instance 1

In this section, we will apply the Laplace–Pad´e differential transform method, which means the modified differential transform method, to find approximate analytical solutions for equal system (5), and further simulate to illustrate the effectiveness of the controller in Section 5.2.

First of all, let us look at a real case (Fig. 1).

S. anglica, native plant of the south coast of England, grows at beach of the rivers and sea. It acts important role of prevention of wave and solidification of embankment. In the sixty's of last century, our country introduced the alien species. However, *S. anglica* spread and propagated on the beach, which has led to some

Fig. 1. Garden workers clean *Spartina anglica*.

unexpected consequences. What is worse is that fish, algae, shellfish and small fish cannot live at beach of *S. anglica*. Since the landscape along with Yongjiang is an estuary, many of the villagers were engaged in aquaculture a few years ago. Now there are almost no left villagers to do this work. On November 4th, management department of Zhenhai District, Ningbo city organized a dozen garden workers, clean *S. anglica* along the Yongjiang. The *S. anglica* threatens to fishery development, and this suppression of *S. anglica* is not a special example. In 2011, Xiangshan County regarded *S. anglica* treatment as one of the hundreds of major projects, and invested in five hundred millions yuan. *S. anglica* was administered in the disaster area by the combining mechanical excavation manual removal method. The above information is from [43].

Extracting the data in above case and further combining the data from Qingdao ocean data sharing platform [44], we can get the parameters values below:

$r=0.5,\ K=20,\ \beta=0.1,\ h=0.2,\ \theta=0.2,\ \omega=0.3,\ \alpha=0.4,\ p=0.1,\ c=0.03$. The periodic input disturbance is used to describe when the interference is introduced into the fish population periodically. Here we take

$b_{11}\delta(t)=a+b\ \sin\ at, a=b=A=1,\ b_{12}=0,\ b_{13}=0,\ b_{14}=0,$

then system (5) becomes

$$\begin{cases} \dot{\xi}_1(t)=-\frac{(0.03+0.1\xi_1(t))(0.5\xi_1(t)+0.5\xi_2(t)+20\xi_4(t))}{2}+1+\ \sin\ t \\ \dot{\xi}_2(t)=0.1\xi_2(t)-0.2\xi_3(t) \\ \dot{\xi}_3(t)=-0.4\xi_1(t)+0.2\xi_2(t)-0.3\xi_3(t) \\ \dot{\xi}_4(t)=0.1\xi_1(t)(0.4325+\xi_4(t)) \end{cases} \tag{14}$$

with the initial conditions

$$\xi_1(0)=0,\ \xi_2(0)=0,\ \xi_3(0)=0,\ \xi_4(0)=0. \tag{15}$$

Taking the differential transform of (14) and (15) respectively for the procedures of the Laplace–Pad´e differential transform method in the appendix, we get

$$\zeta_1(0)=0,\ \zeta_2(0)=0,\ \zeta_3(0)=0,\ \zeta_4(0)=0. \tag{16}$$

And the recursion system

$$\begin{cases} (k+1)\zeta_1(k+1)=-0.0075\zeta_1(k)-0.0075\zeta_2(k)-0.3\zeta_4(k)-0.025\sum_{r=0}^{k}\zeta_1(k)\zeta_1(k-r)-0.025\sum_{r=0}^{k}\zeta_1(k)\zeta_2(k-r)-\sum_{r=0}^{k}\zeta_1(k)\zeta_4(k-r)+\delta(k)+\frac{1}{k!}\ \sin\left(\frac{k\pi}{2}\right) \\ (k+1)\zeta_2(k+1)=0.1\zeta_2(k)-0.2\zeta_3(k) \\ (k+1)\zeta_3(k+1)=-0.4\zeta_1(k)+0.2\zeta_2(k)-0.3\zeta_3(k) \\ (k+1)\zeta_4(k+1)=0.04325\zeta_1(k)+0.1\sum_{r=0}^{k}\zeta_1(k)\zeta_4(k-r) \end{cases}$$

$$\text{for } k=0,\ 1,\ 2,\ \cdots \tag{17}$$

Utilizing (16) and (17), we calculate the first several terms

$\zeta_1(1)=1,\ \zeta_2(1)=0,\ \zeta_3(1)=0,\ \zeta_4(1)=0,$
$\zeta_1(2)=0.48375,\ \zeta_2(2)=0,\ \zeta_3(2)=-0.2,\ \zeta_4(2)=0.021625,$
$\zeta_1(3)=-0.0128403,\ \zeta_2(3)=0.0133333,\ \zeta_3(3)=-0.0445,$
$\zeta_4(3)=0.00732277, \zeta_1(4)=-0.0420048,\ \zeta_2(4)=0.00255833,$
$\zeta_3(4)=0.0052882,\ \zeta_4(4)=-0.000148128,$
$\zeta_1(5)=0.000613444,\ \zeta_2(5)=-0.000160361,\ \zeta_3(5)=0.00314543,$
$\zeta_4(5)=-0.000387536, \zeta_1(6)=0.0014011,\ \zeta_2(6)=-0.00010752,$
$\zeta_3(6)=-0.000203513,\ \zeta_4(6)=4.7124\times10^{-6},$
$\zeta_1(7)=-0.0000145142,\ \zeta_2(7)=4.27866\times10^{-6},$
$\zeta_3(7)=-0.0000744129,\ \zeta_4(7)=9.22558\times10^{-6},$
$\zeta_1(8)=-2.50208\times10^{-5},\ \zeta_2(8)=1.91381\times10^{-6},$
$\zeta_3(8)=3.62316\times10^{-6},\ \zeta_4(8)=-8.36247\times10^{-8}.$

Combining above results to $u_*(t)=\sum_{k=0}^{n}U(k)t^k$ in the appendix, we obtain the eight order solution approximations

$$\begin{aligned} \xi_1(t)\cong\sum_{k=0}^{8}\zeta_1(k)t^k &= t+0.48375t^2-0.0128403t^3-0.0420048t^4 \\ &+0.000613444t^5+0.0014011t^6-0.0000145142t^7 \\ &-2.50208\times10^{-5}t^8, \end{aligned}$$

$$\begin{aligned} \xi_2(t)\cong\sum_{k=0}^{8}\zeta_2(k)t^k &= 0.0133333t^3+0.00255833t^4 \\ &-0.000160361t^5-0.00010752t^6+4.27866\times10^{-6}t^7 \\ &+1.91381\times10^{-6}t^8, \end{aligned}$$

$$\begin{aligned} \xi_3(t)\cong\sum_{k=0}^{8}\zeta_3(k)t^k &= -0.2t^2-0.0445t^3+0.0052882t^4 \\ &+0.00314543t^5-0.000203513t^6-0.0000744129t^7 \\ &+3.62316\times10^{-6}t^8, \end{aligned}$$

$$\begin{aligned} \xi_4(t)\cong\sum_{k=0}^{8}\zeta_4(k)t^k &= 0.021625t^2+0.00732277t^3 \\ &-0.000148128t^4-0.000387536t^5 \\ &+4.7124\times10^{-6}t^6+9.22558\times10^{-6}t^7-8.36247\times10^{-8}t^8. \end{aligned}$$

The solutions series obtained via the differential transform method may have limited regions of convergence. We can apply the Laplace–Pad´e resummation method to improve the accuracy. First, we apply t Laplace transforms to the eight order solution approximations. Second, we substitute s by $1/t$. Then t Pad´e approximants are applied to the transformed series. Finally, we substitute t by $1/s$ and apply the inverse Laplace s transforms to the solution.

Applying t Laplace transforms to the eight order solution approximations,

$$\begin{aligned} \psi[\xi_1(t)] &= \frac{1}{s^2}+\frac{0.9675}{s^3}-\frac{0.0770418}{s^4}-\frac{1.0081152}{s^5}+\frac{0.07361328}{s^6} \\ &+\frac{1.008792}{s^7}-\frac{0.073151568}{s^8}-\frac{1.008838656}{s^9} \end{aligned}$$

$$\begin{aligned} \psi[\xi_2(t)] &= \frac{0.0799998}{s^4}+\frac{0.0613999}{s^5}-\frac{0.01924332}{s^6} \\ &-\frac{0.0774144}{s^7}+\frac{0.0215644464}{s^8}+\frac{0.0771648192}{s^9} \end{aligned}$$

$$\begin{aligned} \psi[\xi_3(t)] &= -\frac{0.4}{s^3}-\frac{0.267}{s^4}+\frac{0.1269168}{s^5}+\frac{0.3774516}{s^6} \\ &-\frac{0.14652936}{s^7}-\frac{0.375041016}{s^8}+\frac{0.1460858112}{s^9} \end{aligned}$$

$$\begin{aligned} \psi[\xi_4(t)] &= \frac{0.04325}{s^3}+\frac{0.04393662}{s^4}-\frac{0.00355507}{s^5}-\frac{0.04650432}{s^6} \\ &+\frac{0.00339293}{s^7}+\frac{0.0464969}{s^8}-\frac{0.00337175}{s^9}. \end{aligned}$$

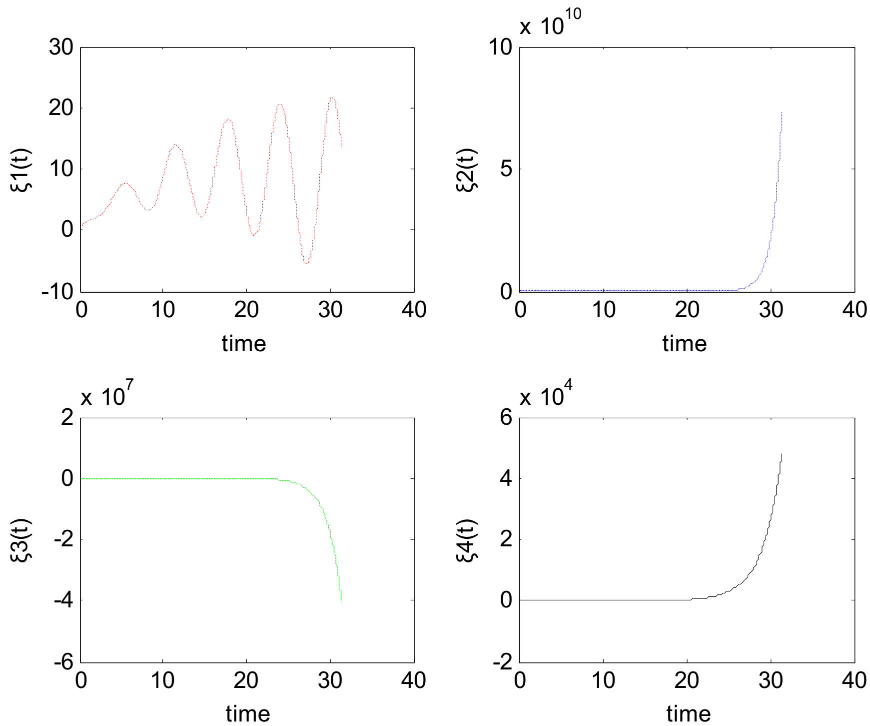

Fig. 2. The Laplace–Pad´e differential transformation method approximations for $\xi_1, \xi_2, \xi_3, \xi_4$.

For the sake of simplicity we assume $s = 1/t$ in the above expressions to obtain

$$\psi[\xi_1(t)] = t^2 + 0.9675t^3 - 0.0770418t^4 - 1.0081152t^5 + 0.07361328t^6 + 1.008792t^7 - 0.073151568t^8 - 1.008838656t^9$$

$$\psi[\xi_2(t)] = 0.0799998t^4 + 0.0613999t^5 - 0.01924332t^6 - 0.0774144t^7 + 0.0215644464t^8 + 0.0771648192t^9$$

$$\psi[\xi_3(t)] = -0.4t^3 - 0.267t^4 + 0.1269168t^5 + 0.3774516t^6 - 0.14652936t^7 - 0.375041016t^8 + 0.1460858112t^9$$

$$\psi[\xi_4(t)] = 0.04325t^3 + 0.04393662t^4 - 0.00355507t^5 - 0.04650432t^6 + 0.00339293t^7 + 0.0464969t^8 - 0.00337175t^9.$$

From above expressions we calculate t Pad´e approximants [4/4], [4/4], [5/4] and [5/4] of $\psi[\xi_1(t)], \psi[\xi_2(t)], \psi[\xi_3(t)]$ and $\psi[\xi_4(t)]$, hence it is obtained that:

$$\left[\frac{4}{4}\right]_{\xi_1} = (t^2 + 1.00552t^3 + 0.964949t^4) \times (1 + 0.0380164t + 1.00521t^2 + 0.0385057t^3 + 0.00490038t^4)^{-1}$$

$$\left[\frac{4}{4}\right]_{\xi_2} = (0.0799998t^4) \times (1 - 0.767501t + 0.829599t^2 + 0.146348t^3 - 0.925022t^4)^{-1}$$

$$\left[\frac{5}{4}\right]_{\xi_3} = (-0.4t^3 - 0.310158t^4 + 1.68 \times 10^{-7}t^5) \times (1 + 0.107896t + 0.245271t^2 + 0.814145t^3 - 0.730129t^4)^{-1}$$

$$\left[\frac{5}{4}\right]_{\xi_4} = (0.04325t^3 + 0.152028t^4 + 3.73803 \times 10^{-7}t^5) \times (1 + 2.49923t - 2.4567t^2 + 3.77638t^3 - 1.42944t^4)^{-1}.$$

Since $t = 1/s$, we get $[4/4]_{\xi_1}$, $[4/4]_{\xi_2}$, $[5/4]_{\xi_3}$ and $[5/4]_{\xi_4}$ in terms of s as follows,

$$\left[\frac{4}{4}\right]_{\xi_1} = (s^2 + 1.00552s + 0.964949) \times (s^4 + 0.0380164s^3 + 1.00521s^2 + 0.0385057s + 0.00490038)^{-1}$$

$$\left[\frac{4}{4}\right]_{\xi_2} = (0.0799998) \times (s^4 - 0.767501s^3 + 0.829599s^2 + 0.146348s^1 - 0.925022)^{-1}$$

$$\left[\frac{5}{4}\right]_{\xi_3} = (-0.4s^2 - 0.310158s + 1.68 \times 10^{-7}) \times (s^5 + 0.107896s^4 + 0.245271s^3 + 0.814145s^2 - 0.730129s)^{-1}$$

$$\left[\frac{5}{4}\right]_{\xi_3} = (0.04325s^2 + 0.152028s + 3.73803 \times 10^{-7}) \times (s^5 + 2.49923s^4 - 2.4567s^3 + 3.77638s^2 - 1.42944s)^{-1}.$$

Finally, applying the inverse s Laplace transforms to the above Pad´e approximants, we obtain the following approximate solutions:

$$\xi_1(t) = e^{(-0.0192477 - 0.0672926i)t}((0.503849 + 7.02059i) + (0.503849 - 7.02059i)e^{(0.134585i)t}) + e^{(-1.00016i)t}((-0.503849 + 0.0373785i)e^{0.000239478t} - (0.503849 + 0.0373785i)e^{(0.000239478 + 2.00033i)t})$$

$$\xi_2(t) = 0.0799998(-0.258444e^{-0.731868t} + 0.36271e^{0.909084t} + e^{(-1.14158i)t}((-0.0521329 - 0.213786i)e^{0.295143t} - (0.0521329 - 0.213786i)e^{(0.295143 + 2.28317i)t}))$$

$$\xi_3(t) = -2.30096 \times 10^{-7} - 0.0278766e^{-1.1018t} - 0.263262e^{0.60021t} + e^{(-1.03214i)t}((0.14557 + 0.0339043i)e^{0.196848t} + (0.14557 - 0.0339043i)e^{(0.196848 + 2.06428i)t})$$

$$\xi_4(t) = -2.61503 \times 10^{-7} + 0.0000108605e^{-3.53053t} + 0.0503052e^{0.437965t} + e^{(-0.914575i)t}((-0.0251579 - 0.00386332i)e^{0.29666t} - (0.0251579 - 0.00386332i)e^{(0.29666 + 1.82915i)t}).$$

Fig. 2. shows the Laplace–Pad´e differential transformation method approximations for model (5). It demonstrates that as the fish population fluctuates gradually in the vicinity of the

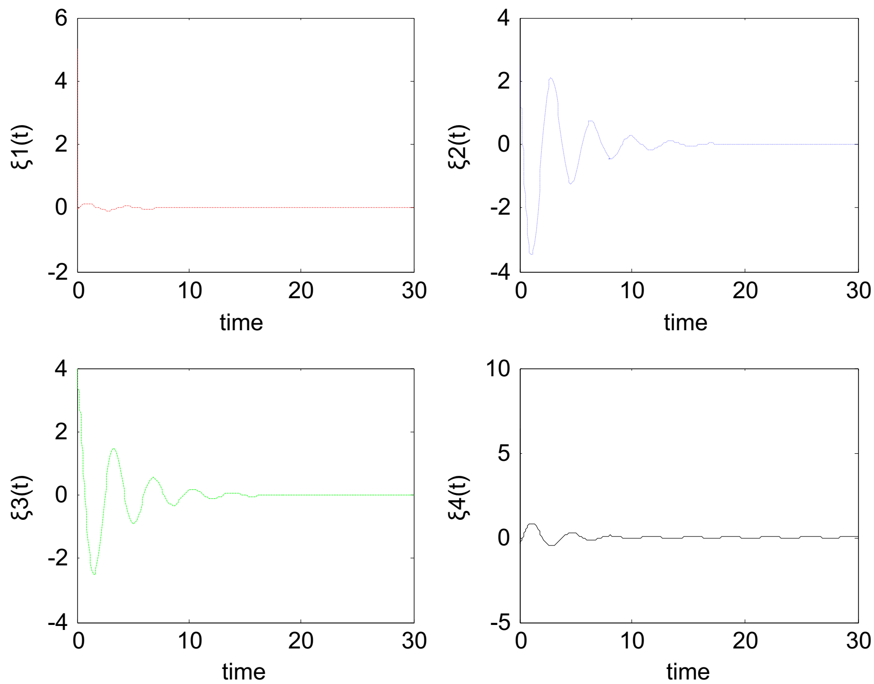

Fig. 3. Responses of $\xi_1(t), \xi_2(t), \xi_3(t), \xi_4(t)$ in system (12).

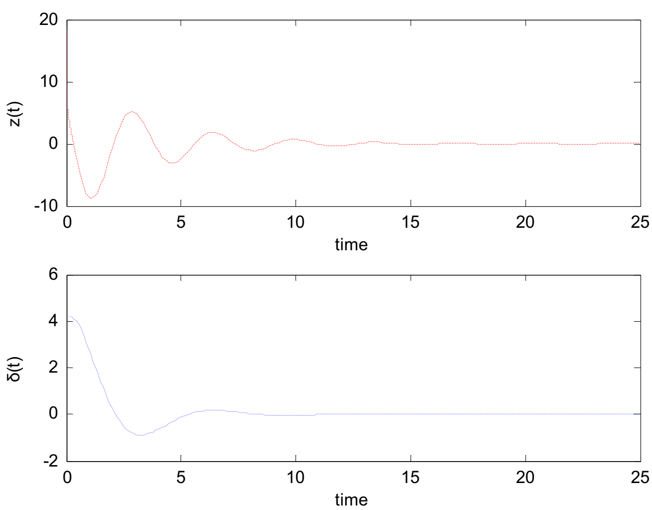

Fig. 4. Responses of $z(t), \delta(t)$ in system (12) $\gamma = 0.05$.

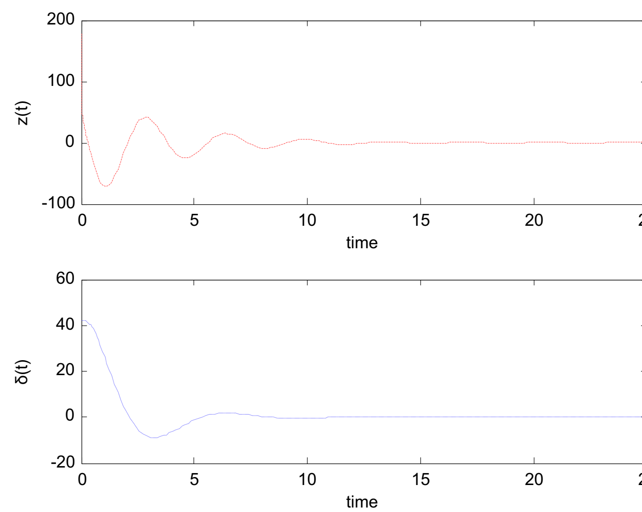

Fig. 5. Responses of $z(t), \delta(t)$ in system (12) $\gamma = 0.005$.

equilibrium, the harvest capacity increases. Simultaneously, density of *S. anglica* population increases from the equilibrium exponentially in a certain period, therefore, the purification capacity decreases from the equilibrium relatively.

5.2. Instance 2

In this section, we further simulate to illustrate the effectiveness of H_∞controller. Also using the above data, we further properly process it (non-dimensional transformation, equal ratio simplification, approximation and so on) according to the parameters in the model of this paper. In sequence, the values of parameters in system (6) are as follows:

$$r = 1,\ K = 4,\ \beta = 1.5,\ h = 3,\ \theta = 2,\ \omega = 2,\ \alpha = 1,\ p = 1,$$
$$c = 40,\ d = 10,\ b_{11} = 0.1.$$

We design H_∞controller for system (6) so that H_∞ norm of closed loop system (12) is less than $\gamma = 0.05$. Based on Theorem 2 and the LMI toolbox of Matlab, we can obtain the parameters of the controller satisfying the conditions of theorem as follows:

$$K_1 = |7.46962.78830.379822.9737|,$$
$$K_2 = |-2.44062.18320.10433.9878|.$$

Fig. 3 describes that state variables of system (12) trend gradually to zero as time goes on, which is the equilibrium of the original model. Therefore the closed-loop system (12) reaches the steady state. Fig. 4 shows that the output and disturbance input of the closed loop system (12) tend to zero as time goes, which also the equilibrium of model before transformation. This fully demonstrates the feasibility of the controller.

Here the implication of γ for biological significance is the positivity and urgency of government management. We may as well adjust γ to 0.005, which means the relative loose of the government on fisheries, aquaculture and pollution purification management. As it can be seen from Fig. 5, the fluctuation range of the input disturbance and output of the system greatly increased with decrease of γ. This has a detrimental effect for maintaining ecological balance and protect environment. It is worth mentioning that the controller is still effective in making the system a stable state.

Summarizing the information from Figs. 2–5 given above, we can conclude that the unique features of the approaches proposed and the main advantages of the results over others are the authenticity of system description and effectiveness of the H_∞ controller. H_∞ control, which we apply to the biological model in this paper, cannot only effectively control the bifurcation but also guarantee the co-existence of fish population and alien species *S. anglica* in a certain range. Besides, the proposition of purification capacity provides a new idea to research the alien species in biological mathematics.

6. Conclusion

The problem of H_∞ control for fish population logistic model with the invasion of alien species has been investigated by means of the T–S fuzzy system approach in this paper. The method that combines purification capacity and H_∞ control is applied to relieve the stress on environmental resources and ecosystems. The elimination of the singular induced bifurcation and the satisfaction of the norm of H_∞ control can be achieved via constructing the T–S fuzzy system. Finally, the simulation results for the model have been provided to validate the effectiveness of the method used in this paper and these illustrations have clearly demonstrated that H_∞ controller can improve the performance of the system.

From a biological perspective, new management approaches must be considered to stem the damage and ensure that the fisheries and aquaculture ecosystems, as well as their unique features are protected. Current management of fish resources is insufficient. Since *S. anglica* can promote the sedimentation and reclamation, increasing interest has been stimulated to study it. The character of soil can be improved so that the environment can be protected. In consequence, it is essential to control the density of *S. anglica*. The H_∞ control approach we proposed in this paper cannot only effectively control the bifurcation but also guarantee the co-existence of fish population and alien species *S. anglica* in a certain range. At the same time, adjusting the H_∞ norm γ effectively controls the input disturbance and output in concessional limit of environmental. Hence, the results of our study generate a profound impact on better developing the relationship of alien species and fishery.

In future work, we will research the issue that if it is possible to extend the current results for the underlying systems under the network-based environment with time-delays, packet dropouts, and quantization.

Table 1
Main operations of differential transform method.

Function	*differential_transform*
$\alpha u(t) \pm \beta v(t)$	$\alpha U(k) \pm \beta V(k)$
$u(t)v(t)$	$\sum_{r=0}^{k} U(k)V(k-r)$
$\frac{d}{dt}[u(t)]$	$(k+1)U(k+1)$
t^n	$\delta(k-n) = \begin{cases} 1, & k=n \\ 0, & k \neq n \end{cases}$
$\sin(\omega t+\alpha)$	$\frac{\omega^k}{k!} \sin(\frac{\pi k}{2}+\alpha)$

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Appendix

The introduction to the Laplace–Pad´e differential transform method is as follows.

Firstly, the basic definition and fundamental operations of the differential transform method are introduced briefly. Based on the definitions of the differential transform given in [41], we give a review of the method for convenience. Therefore we obtain the modified differential transform method naturally.

Definition 1. If a function $u(t)$ is analytical with respect to t in the domain of interest Ω, then

$$U(k) = \frac{1}{k!}\left[\frac{d^k u(t)}{dt^k}\right]_{t=0}$$

is the transformed function of $u(t)$.

Definition 2. The differential inverse transform of the set $\{U(k)\}_{k=0}^{n}$ is defined by:

$$u(t) = \sum_{k=0}^{\infty} U(k)t^k.$$

Simplify the two equations above, one deduces that

$$u(t) = \sum_{k=0}^{\infty} \frac{1}{k!}\left[\frac{d^k u(t)}{dt^k}\right]_{t=0} t^k.$$

From Definitions 1 and 2, it is easy to see that the concept of the differential transform method is obtained from the power series expansion. Considering the nonlinear system

$$\frac{du(t)}{dt} = f(u(t), t), \quad t \geq 0,$$

where $f(u(t), t)$ is a nonlinear smooth function. The system above is supplied with some initial conditions

$$u(0) = u_0.$$

The differential transform method discussed above establishes that the solution can be written as:

$$u(t) = \sum_{k=0}^{\infty} U(k)t^k.$$

Where $U(0), U(1), U(2), \ldots$ are unknowns to be determined by the differential transform method. Applying the differential transform method to the initial condition and the system, respectively, we obtain the transformed initial condition

$$U(0) = u_0,$$

and the recursion system

$$(1+k)U(1+k) = F(U(0), \ldots U(k), k), k = 0, 1, 2, \ldots,$$

where $F(U(0), \ldots, U(k), k)$ is the differential transform of $f(u(t), t)$. Using the two equations above, we determine the unknowns $U(k), k = 0, 1, 2, \ldots,$then the differential inverse transformation of the set of values $\{U(k)\}_{k=0}^{n}$ gives the approximate solution

$$u_*(t)=\sum_{k=0}^{n}U(k)t^k$$

where n is the approximation order of the solution. The exact solution of problem $\frac{du(t)}{dt}=f(u(t),t), t\geq 0$ and $u(0)=u_0$ is then given by:

$$u(t)=\sum_{k=0}^{\infty}U(k)t^k.$$

If $U(k)$ and $V(k)$ are the differential transforms of $u(t)$ and $v(t)$ respectively, then the main operations of differential transform method are shown in Table 1.

The process of differential transform method can be summarized as follows.

(1) Apply the differential transform to the initial condition $u(0)=u_0$.
(2) Apply the differential transform to the differential system $du(t)/dt=f(u(t),t), t\geq 0$ to obtain a recursion system for the unknowns $U(0), U(1), U(2), \ldots$
(3) Use the transformed initial condition $U(0)=u_0$ and the recursion system $(1+k)U(1+k)=F(U(0),\ldots U(k),k), k=0,1,2,\ldots,$ to determine the unknowns $U(0), U(1), U(2), \ldots$
(4) Use the differential inverse transform formula $u_*(t)=\sum_{k=0}^{n}U(k)t^k$ to obtain an approximate solution for the initial value problem $du(t)/dt=f(u(t),t), t\geq 0$ and $u(0)=u_0$.

The solutions series obtained from differential transform method may have limited region of convergence, even if we take a large number of terms. Therefore, we propose to apply the Laplace–Pad´e resummation method to differential transform method truncated series to enlarge the convergence region. Considering Maclaurin's expansion of analytical function $u(t)=\sum_{n=0}^{\infty}u_nt^n, 0\leq t\leq T$. The Pad´e approximation to $u(t)$ of order $[L,M]$ which we denote by $[L/M]_u(t)$ is defined by [42]:

$$\left[\frac{L}{M}\right]_u(t)=\frac{p_0+p_1t+\cdots+p_Lt^L}{1+q_1t+\cdots+q_Mt^M}$$

We considered $q_0=1$, and the numerator and denominator have no common factor. The numerator and denominator are constructed so that $u(t)$,$[L/M]_u(t)$ and their derivatives agree at $t=0$ up to $L+M$. That is,

$$u(t)-\left[\frac{L}{M}\right]_u(t)=O\left(t^{L+M+1}\right).$$

From the equation above, we have

$$u(t)\sum_{n=0}^{M}q_nt^n-\sum_{n=0}^{L}p_nt^n=O\left(t^{L+M+1}\right).$$

Therefore, we get the following algebraic linear systems:

$$\begin{aligned}&u_Lq_1+\cdots+u_{L-M+1}q_M=-u_{L+1}\\&u_{L+1}q_1+\cdots+u_{L-M+2}q_M=-u_{L+2}\cdots\\&u_{L+M-1}q_1+\cdots+u_Lq_M=-u_{L+M},\\&p_0=u_0\\&p_1=u_1+u_0q_1\ldots\\&p_L=u_L+u_{L-1}q_1+\cdots+u_0q_1.\end{aligned}$$

From the equations above, we calculate first all the coefficients $q_n, 1\leq n\leq M$. Then, we determine the coefficients $p_n, 0\leq n\leq L$. Note that for a fixed value of $L+M+1$, $u(t)-[L/M]_u\ (t)=O\left(t^{L+M+1}\right)$ is smallest when the numerator and denominator of $[L/M]_u(t)=p_0+p_1t+\cdots+p_Lt^L/1+q_1t+\cdots+q_Mt^M$ have the same degree or when the numerator has degree one higher than the denominator.

Several approximate methods provide power series solution. However, sometimes, this type of solutions lack of large domain of convergence. Therefore, the Laplace–Pad´e resummation method is used to enlarge the convergent domain of the solution to find the exact solution. The Laplace–Pad´e resummation method can be described as follows.

(1) First, Laplace transformation is applied to power series $u_*(t)=\sum_{k=0}^{n}U(k)t^k$.
(2) Next,s is substituted by $1/t$ in the resulting equation.
(3) Then, we convert the transformed series into a meromorphic function by forming its Pad´e approximant of order $[N/M]$. N and $M$are chosen randomly, but they should be smaller than the order of the power series. In this step, the Pad´e approximant extends the domain of the truncated series solution to obtain a better accuracy and convergence.
(4) After that, t is substituted by $1/s$.
(5) At last, we obtain the exact or approximate solution via the inverse Laplace $s$transformation.

Now, we will apply the Laplace–Pad´e differential transform method, which means the modified differential transform method, to find approximate analytical solutions for the equal system (5), and further illustrate the effectiveness of the controller in Section 5.2.

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