A family of quantum information theory inspired metrics for unitary matrices

H. F. Chau
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Abstract
I present my recent work [arXiv:1006.3614] on a family of quantum information theory inspired metrics for unitary matrices. These metrics give meaningful measure on the degree of non-commutativity between two unitary matrices. Furthermore, several new inequalities involving products of unitary matrices are discovered as by-products.

Characterization of affine automorphisms and ortho-order automorphisms of quantum probabilistic maps

Shuanping Du
Xiamen University

Abstract
In quantum mechanics, often it is important for the representation of quantum system to study the structure-preserving bijective maps of the quantum system. Such maps are also called isomorphisms or automorphisms. In this note, using the Uhlhorn-type of Wigner's theorem, we characterize all affine automorphisms and ortho-order automorphisms of quantum probabilistic maps.

Co-author Zhaofang Bai (Xiamen University)

Universal squash model for optical communications using linear optics and threshold detectors

Chi-Hang Fred Fung
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Abstract
Quantum communications often rely on single photons as information carriers in order to exploit their quantum mechanical properties. However, practical detectors are often threshold detectors that are incapable of resolving the number of photons received. This apparently subtle issue has surprisingly immense implication to many quantum communications protocols. In fact, it has been shown that this issue leads to many problems including fake violation of Bell's inequality, insecurity of quantum key distribution, and false entanglement verification. The source of these problems is the discrepancy between the theoretical

consideration where single-photon signals are assumed and the actual experiments where multi-photon signals may be detected. We report a universal solution to bridge this gap between theory and experiments.

Co-authors H. F. Chau (The University of Hong Kong) and Hoi-Kwong Lo (University of Toronto)

The CCNR criterion of separability for states in infinite-dimensional quantum systems

Yu Guo
Shanxi University

Abstract
In this paper, the realignment criterion and the CCNR criterion of separability for states in infinite-dimensional bipartite quantum systems are established. Let H_A and H_B be complex Hilbert spaces with $\dim H_A \otimes H_B = +\infty$. Let ρ be a state on $H_A \otimes H_B$ and $\{\delta_k\}$ be the Schmidt coefficients of ρ as a vector in the Hilbert space $\mathcal{C}_2(H_A) \otimes \mathcal{C}_2(H_B)$. We introduce the realignment operation ρ^R and the computable cross norm $\|\rho\|_{\mathrm{CCN}}$ of ρ and show that, if ρ is separable, then $\|\rho^R\|_{\mathrm{Tr}} = \|\rho\|_{\mathrm{CCN}} = \sum_k \delta_k \leq 1$. In particular, if ρ is a pure state, then ρ is separable if and only if $\|\rho^R\|_{\mathrm{Tr}} = \|\rho\|_{\mathrm{CCN}} = \sum_k \delta_k = 1$.

Co-author Jin-Chuan Hou (Taiyuan University of Technology)

Characterizing sequential isomorphisms on Hilbert space effect algebras

Kan He
Taiyuan University of Technology

Abstract
Let $\star$ be any sequential product on the Hilbert space effect algebra $\mathcal{E}(H)$ with $\dim H \geq 2$ and $\Phi : \mathcal{E}(H) \to \mathcal{E}(H)$ be a bijective map. We show that if Φ satisfies that $\Phi(A \star B) = \Phi(A) \star \Phi(B)$ for $A, B \in \mathcal{E}(H)$, then there is an either unitary or anti-unitary operator U such that $\Phi(A) = UAU^\dagger$ for every $A \in \mathcal{E}(H)$. Let $g : [0,1] \to \{\lambda \mid \lambda \in \mathbb{C}, |\lambda| = 0 \text{ or } 1\}$ be a Borel function satisfying $g(0) = 0, g(1) = 1$ and define a binary operation $\diamond_g$ on $\mathcal{E}(H)$ by $A \diamond_g B = A^{1/2} g(A) B g(A)^\dagger A^{1/2}$, where $T^\dagger$ denotes the conjugate of the operator T. We also show that a bijective map $\Phi : \mathcal{E}(H) \to \mathcal{E}(H)$ satisfies that $\Phi(A \diamond_g B) = \Phi(A) \diamond_g \Phi(B)$ for

$A, B \in \mathcal{E}(H)$ if and only if there is an either unitary or anti-unitary operator U such that $\Phi(A) = UAU^\dagger$ for every $A \in \mathcal{E}(H)$.

Co-authors Jinchuan Hou (Taiyuan University of Technology) and Xiaofei Qi (Shanxi University)

When different entanglement witnesses detect entangled states simultaneously

Jin-Chuan Hou
Taiyuan University of Technology

Abstract
The question under what conditions different witnesses may detect some entangled states simultaneously is answered for both finite- and infinite-dimensional bipartite systems. Finite many different witnesses can detect some entangled states simultaneously if and only if their convex combinations are still witnesses; they can not detect any entangled state simultaneously if and only if the set of their convex combinations contains a positive operator. For two witnesses W_1 and W_2, some more can be said: (1) W_1 and W_2 can detect the same set of entangled states if and only if they are linearly dependent; (2) W_2 can detect more entangled states than that W_1 can if and only if W_1 is a linear combination of W_2 and a positive operator. As an application, some characterizations of the optimal witnesses are given and some structure properties of the decomposable optimal witnesses are presented.

Co-author Yu Guo (Shanxi University)

Entanglement entropy via string theory

Ling-Yan Hung
Perimeter Institute

Abstract
We briefly review the AdS/CFT correspondence discovered in string theory over a decade ago and its application to the computation of entanglement entropy in a conformal field theory, where a quantum field theory problem is converted to a geometrical problem. Non-trivial matches with field theory computations have been found in the case of 1+1 d CFT. We will discuss recent progress in extending the proposal to include CFTs of general central charges, and finding quantitative matches with higher dimensional CFTs.

Co-authors Rob Myers and Michael Smolkin

The automorphism group of separable states in quantum information theory

Chi-Kwong Li
College of William and Mary

Abstract

We show that the linear group of automorphism of Hermitian matrices which preserves the set of separable states is generated by *natural* automorphisms: change of an orthonormal basis in each tensor factor, partial transpose in each tensor factor, and interchanging two tensor factors of the same dimension. We apply our results to preservers of the product numerical range.

Co-authors Shmuel Friedland (University of Illinois at Chicago), Yiu-Tung Poon (Iowa State University), and Nung-Sing Sze (The Hong Kong Polytechnic University)

Insecurity of position-based quantum cryptography protocols against entanglement attacks

Hoi-Kwong Lo
University of Toronto

Abstract

Recently, position-based quantum cryptography has been claimed to be unconditionally secure. On the contrary, here we show that the existing proposals for position-based quantum cryptography are, in fact, insecure if entanglement is shared among two adversaries. Specifically, we demonstrate how the adversaries can incorporate ideas of quantum teleportation and quantum secret sharing to compromise the security with certainty. The common flaw to all current protocols is that the Pauli operators always map a codeword to a codeword (up to an irrelevant overall phase). We propose a modified scheme lacking this property in which the same cheating strategy used to undermine the previous protocols can succeed with a rate at most 85modified protocol is secure when the shared quantum resource between the adversaries is a two- or three- level system. Reference: http://arxiv.org/abs/1009.2256

Co-author Hoi-Kwan Lau (University of Toronto)

Hamiltonian determination through edge spin in transverse field ising chain

Mikio Nakahara
Kinki University

Abstract

Fixing parameters in a Hamiltonian with a good precision is an important aspect of quantum information processing and quantum control. Without knowing the Hamiltonian, it is impossible to control the system at our disposal.

It may happen that we have an access to only a single qubit in a quantum register. This is the case when the rest of the register is isolated from the environment to suppress decoherence, for example. Then we have to estimate the Hamiltonian through this particular qubit.

Burgarth, Maruyama and Nori[**?**, **?**] have studied this problem in the Heisenberg chain and XXZ chains, for which the z-component $\sum_{i=1}^{N}\sigma_z^i$ of the total spin is conserved. Here σ_k^i denotes the k-th Pauli matrix at the i-th site. Then the first excited state is a superposition of N basis vectors, which makes the Hamiltonian estimation easy. In this talk, we condier the transverse field Ising chain with the Hamiltonian

$$H = \sum h_i\sigma_x^i + \sum_{i=1}^{N-1} J_{i,i+1}\sigma_z^i\sigma_z^{i+1},$$

which has no such conserved quantities and the excited states involve all the 2^N basis vectors.[**?**] This is also studied by Burgarth, Maruyama and Nori in a different formalism.[**?**]

We do the case study of this system with $N = 3$ in this talk. We evaluate the time-evolution operator $U(t) = e^{-iHt}$ exactly and obtain the expectation value $\langle\psi(0)|U(t)^\dagger\sigma_x^1U(t)|\psi(0)\rangle$ and its Fourier transformation, where we take $|\psi(0)\rangle = |000\rangle$ when $h_1 \neq 0$ and $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)|00\rangle$ for $h_1 = 0$. By assumption, we can control and measure the first qubit only. It is shown that we need to evaluate the Fourier transformation for a few h_1 to determine the other parameters h_2, h_3, J_{12} and J_{23} completely.

Extension of this work to an arbitrary N-chain will be reported.

Co-authors Shu Tanaka, Yasushi Kondo, and Mohammad Ali Fasihi

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Implementation of QECC with NMR quantum computer

Mikio Nakahara
Kinki University

Abstract

We propose an implementation of a 3-qubit bit-flip QECC proposed in [1] using an NMR quantum computer. A qubit state $|\psi\rangle = a|0\rangle + b|1\rangle$ is encoded as $|\Psi\rangle = a|000\rangle + b|111\rangle$ and sent through a noisy channel. It is assumed that the bit-flip operator X acts on one of the qubits during transmission.

We show that the recovery operation can be implemented with a unitary quantum circuit, which contains several CNOT gates. The rank 2 projection operator P required for QECC is effectively implemented by a permutation of the qubit basis vectors and ignoring (i.e., filter out) unnecessary NMR signals from the free induction decay signal.

We also suggest a new non-unitary process, which replaces the projection operator, with which our technique can be applied to 9-qubit and 5-qubit QECC.

Co-authors Hiroyuki Tomita (Kinki University) and Yasushi Kondo (Kinki University)

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Beyond realignment criterion

Yiu-Tung Poon
Iowa State University

Abstract

Suppose there are two quantum systems described by states in two Hilbert spaces $\mathcal{H}_1$ and $\mathcal{H}_2$ with dimension m and n, respectively. An important problem in quantum information science is to determine if a density operator ρ on $\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2$ is separable. Lupo, Aniello, and Scardicchio proposed a criterion based on the singular values $\sigma(\rho^R)$ of the realignment matrix ρ^R of ρ. We obtain bounds for the elementary symmetric functions on $\sigma(\rho^R)$ and show that the criterion is the same as the realignment (CCNR) criterion when $m = n$.

Co-authors Chi-Kwong Li (College of William and Mary) and Nung-Sing Sze (The Hong Kong Polytechnic University)

Detecting entanglement of states by entries of their density matrices

Xiaofei Qi
Shanxi University

Abstract

For any bipartite systems, a universal entanglement witness of rank-4 for pure states is obtained and a class of finite rank entanglement witnesses is constructed. In addition, a method of detecting entanglement of a state only by entries of its density matrix with respect to some product basis is obtained.

Co-author Jin-Chuan Hou (Taiyuan University of Technology)

Characterizing instances for a decision problem of bipartite nonclassical correlation

Akira SaiToh
Kinki University

Abstract

Nonclassical correlation other than entanglement is of growing interest. The problem of deciding if a bipartite quantum system possesses vanishing quantum discords for both sides is equivalent to deciding if its density matrix has an eigenbasis in the form of a product of local eigenbases [1]. The problem is formally defined as follows.

Definition. The vanishing nonclassical correlation problem.

Instance: A bipartite density matrix ρ^{AB} and $\varepsilon > 0$, over rational numbers and/or finite-bit-length floating-point numbers. (The dimensions of the Hilbert spaces for subsystems A and B are d^{A} and d^{B}, respectively.)

Question: Decide if there exists a pair of complete orthonormal systems (CONSs), $\{|a_i\rangle^{\mathrm{A}}\}_{i=1}^{d^{\mathrm{A}}}$ and $\{|b_j\rangle^{\mathrm{B}}\}_{j=1}^{d^{\mathrm{B}}}$, such that

$\|\rho^{\mathrm{AB}}|a_i\rangle^{\mathrm{A}}|b_j\rangle^{\mathrm{B}} - {}^{\mathrm{A}}\langle a_i|^{\mathrm{B}}\langle b_j|\rho^{\mathrm{AB}}|a_i\rangle^{\mathrm{A}}|b_j\rangle^{\mathrm{B}}|a_i\rangle^{\mathrm{A}}|b_j\rangle^{\mathrm{B}}\| \leq \varepsilon \quad \forall\, i,j$, where $\||v\rangle\| = \sqrt{\langle v|v\rangle}$ for vector $|v\rangle$.

There are known cases where an instance becomes an easy instance for at least one of the known methods, which include the cases: (i) ρ^{AB} is nondegenerate, (ii) $\mathrm{Tr}_{\mathrm{B}}\rho^{\mathrm{AB}}$ and $\mathrm{Tr}_{\mathrm{A}}\rho^{\mathrm{AB}}$ are nondegenerate, (iii) either $\mathrm{Tr}_{\mathrm{B}}\rho^{\mathrm{AB}}$ or $\mathrm{Tr}_{\mathrm{A}}\rho^{\mathrm{AB}}$ is nondegenerate [2], (iv) there is a unique operator Schmidt decomposition for ρ^{AB} [3], (v) a particular nonclassical correlation exists and is detectable for one of the nonclassical correlation witnesses [4, 5]. In this short talk, we will discuss easy and hard instances for the problem.

Co-authors Robabeh Rahimi (University of Waterloo) and Mikio Nakahara (Kinki University)

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Realizing and Probing non-Abelian Majorana fermions in ultracold atomic superfluid

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Manipulation of nuclear spins in semiconductor nanostructures

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Abstract

A single electron spin localized in III-V semiconductor structure is an attractive candidate for a solid state quantum bit. An outstanding bottleneck towards spin-based quantum computation has been the fast dephasing of the electron spin by the inevitable nuclear spin environment. In this talk, I will discuss preparations of nuclear spin environment using optical and electrical controlled dynamic nuclear spin polarization to substantially increase the electron spin dephasing time. I will also introduce a novel approach to squeeze the nuclear spin bath into many-body singlets, where the deleterious environmental moments are effectively annihilated. The large scale entanglement in many-body singlets may also become a useful resource for nuclear spin based information processing.
