# THE HONG KONG POLYTECHNIC UNIVERSITY 

Department of Applied Mathematics

## Examination

Subject Code: AMA1501/AMA1502

## Subject Title: Introduction to Statistics for Business / Introduction to Statistics

Session: $\quad$ Semester 2, 2016/2017
Date: 4 May 2017 Time: 8:45 a.m. - 11:45 a.m.
Time Allowed: 3 Hours

This question paper has $\qquad$ 13 pages (attachments included).

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Instructions to Candidates:This question paper has SIX questions.
    Attempt FIVE questions.
    All questions carry equal marks.
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Attachments: Standard Normal Distribution Table, $t$-distribution Table, $\chi^{2}$-distribution Table, F-distribution Table, Formula Sheets

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1. The manager of a bank investigates the percentage of monthly household income used for mortgage payment. A random sample of customers with mortgage loan is selected and the results are tabulated below:

| Percentage of monthly household income | Number of customers |
| :---: | :---: |
| 10 - below 20 | 2 |
| 20 - below 30 | 8 |
| 30 - below 35 | 20 |
| 35 - below 40 | 36 |
| 40 - below 45 | 28 |
| 45 - below 50 | 16 |
| 50 - below 60 | 10 |

(a) Calculate the mean, mode and standard deviation of percentage of household income used for mortgage payment.
(b) Estimate, from the frequency distribution table, the number of customers with percentage of monthly household income used for mortgage payment greater than 42\%.
(4 marks)
(c) The mean and standard deviation of a similar study conducted 10 years ago were $35 \%$ and $15 \%$, respectively. Discuss on which set of data has a larger variability.
(3 marks)
(d) The manager claims that the mean percentage of monthly household income used for mortgage payment of all customers with mortgage loan is not more than $35 \%$. Test for the manager's claim at $2.5 \%$ level of significance.
(6 marks)
2. (a) There are twelve tourist attractions, including 4 museums, 5 historical buildings and 3 natural parks that are specifically promoted by the Tourist Association of City X. Jacky would like to visit three of them in each of the coming two days. If the tourist attractions are selected at random, calculate the number of possible sequence of visiting them when two of each type are to be selected. (4 marks)
(b) The manager of a credit card company studies the behaviour of their credit card holders. From customer records, it is found that $45 \%$ of credit card holders set automatic payment instructions to settle credit card payments and $60 \%$ of credit card holders use Octopus Automatic Add Value Service. Among the users of Octopus Automatic Add Value Service, 55\% of them set automatic payment instructions to settle credit card payments.
(i) Calculate the probability that a randomly selected credit card holder sets automatic payment instructions to settle credit card payments or uses Octopus Automatic Add Value Service.
(4 marks)
(ii) It is known that a randomly selected credit card holder does not set automatic payment instructions to settle credit card payments, calculate the probability that he/she is not a user of Octopus Automatic Add Value Service. (4 marks)
(c) There are three project assistants, Joe, Jenny and Jack, who prepare price proposals to their project manager. From records, $20 \%, 30 \%$ and $50 \%$ of price proposals are prepared by Joe, Jenny and Jack, respectively and their corresponding probability of under-estimating the cost are $0.05,0.02$ and 0.01 .
(i) It is known that a randomly selected price proposal does not under-estimate the cost, calculate the probability that it is prepared by Jenny.
(4 marks)
(ii) Twenty price proposals are chosen randomly and it is found that none of them under-estimates the cost, calculate the probability that six of them are prepared by Jenny.
(4 marks)
3. (a) Time spent by students in handling their final year project is approximately normally distributed with a mean of 220 hours and a standard deviation of 50 hours.
(i) Calculate the probability that a randomly selected student spent between 205 hours and 245 hours in handling his/her final year project.
(3 marks)
(ii) Determine the time that $15 \%$ of students spent more time than it in handling their final year project.
(3 marks)
(iii) Fifty students are selected at random, calculate the probability that at least half of them spent more than 205 hours in handling their final year project.
(6 marks)
(iv) Calculate the probability that the average time spent by sixteen randomly selected students is less than 245 hours.
(4 marks)
(b) The arrival of VIP customers at the customer services counter of a department store follows the Poisson distribution with a mean of 12 per hour. Calculate the probability that in a 15 -minute period, there are more than four VIP customers arriving at the customer services counter.
4. (a) An analyst of a travel agency investigates the preference of customers regarding the duration of tour.
(i) In random samples of 200 adults and 150 elderlies, 120 adults and 50 elderlies, respectively, prefer a tour visiting Country X would have a duration of at most 7 days. Can the analyst conclude that the population proportion of all adults who prefer a tour visiting Country X would have a duration of at most 7 days is higher than that of elderlies at $1 \%$ level of significance?
(ii) Determine the required sample size in estimating the population proportion of all customers who prefer a tour visiting Country Y would have a duration of at most 7 days, with at least $95 \%$ confidence that the estimation error is not more than 0.03 .

## (Question 4 Cont'd)

(b) In a recent utilization survey of an e-learning platform of a tertiary institution, a random sample of 36 students is selected. Their weekly hours in using the elearning platform to perform learning related activities has a mean of 38 hours and a standard deviation of 8 hours. Construct a $95 \%$ confidence interval for the mean weekly hours of all students in using the e-learning platform to perform learning related activities.
(4 marks)
(c) The CEO of ISB Company investigates the views of staff members on the new reward system. Random samples of staff members of management level and junior level are selected to evaluate the new reward system with the statistics given below (higher score for more positive response):

| Staff | Sample size | Mean | Standard deviation |
| :---: | :---: | :---: | :---: |
| Management level | 12 | 68 | 9 |
| Junior level | 18 | 74 | 10 |

Can the CEO conclude that staff members of junior level, on average, have a more positive response to the new reward system than staff members of management level, at $1 \%$ level of significance? State your assumptions.
(6 marks)
5. (a) In comparing the performance of the sales team and customer services team of a computer company, a random sample of customers is selected to evaluate the performances of the two teams. The performance scores of the two teams given by the customers are listed below:

| Customer | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sales team | 70 | 65 | 72 | 80 | 73 | 84 | 69 | 77 |
| Customer services team | 78 | 69 | 70 | 84 | 80 | 90 | 78 | 80 |

Is there any evidence indicating that the mean performance score of sales team is lower than that of customer services team at $5 \%$ level of significance? ( 7 marks)
(b) The manager of a bank studies the arrivals of customers. The following table summarizes the number of customers arriving at the bank for a random sample of 108 non-overlapping 5-minute periods.

| Number of customers | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of periods | 5 | 14 | 25 | 28 | 18 | 12 | 6 |

Test whether the customer arrivals at the bank follows the Poisson distribution at $2.5 \%$ level of significance.
(c) An analyst of the student placement office of an institution studies the association between students' academic performance and employer's evaluation after placement. A random sample of students, who completed the placement in the last semester, is selected and the results are tabulated in the following table:

## (Question 5 Cont'd)

| Academic <br> performance | Employer's evaluation |  |  |
| :---: | :---: | :---: | :---: |
|  | Very satisfactory | Satisfactory | Unsatisfactory |
| Excellent | 42 | 26 | 10 |
| Good | 55 | 69 | 5 |
| Fair | 4 | 24 | 15 |

Test whether students' academic performance and employer's evaluation are independent at $1 \%$ level of significance.
(7 marks)
6. (a) A linear regression equation is proposed to predict monthly electrical usage (y, in kilowatt-hours) on size of apartment ( $x$, in square feet). Summaries of nine randomly selected apartments (with size from 1200 square feet to 2500 square feet) in the last month are:

$$
\begin{aligned}
& \sum x=15870, \quad \sum x^{2}=29170500, \quad \sum y=13993, \quad \sum y^{2}=22458967 \text { and } \\
& \sum x y=25558030
\end{aligned}
$$

(i) Find the least squares linear regression equation for predicting monthly electrical usage on size of apartment.
(4 marks)
(ii) Predict the monthly electrical usage when size of the apartment is 3500 square feet, and discuss its validity.
(b) In estimating the equation of household food consumption in a month (\$0000) on monthly household income (\$0000) and household size, 25 households are selected and the results of regression analysis are provided below:

|  | Coefficients | Standard Error |
| :--- | :---: | ---: |
| Intercept | 1.43260377 | 0.14673751 |
| Monthly household income | 0.00999062 | 0.00316806 |
| Household size | 0.37928986 | 0.02772472 |

Furthermore, $9.84 \%$ of variation in household food consumption in a month is not explained by the fitted model.
(i) Find the fitted multiple linear regression equation.
(ii) Test, at $5 \%$ level of significance, whether monthly household income is a significant independent variable in the proposed model.
(iii) Test whether the overall model is significant at $1 \%$ level of significance.
(iv) Interpret the regression coefficient estimate of the monthly household income.
(2 marks)

