# THE HONG KONG POLYTECHNIC UNIVERSITY 

Department of Applied Mathematics

## Examination

Subject Code: AMA1501/AMA1502
Subject Title: Introduction to Statistics for Business / Introduction to Statistics

Session: $\quad$ Semester 2, 2017/2018
Date: 27 April 2018 Time: 3:15 p.m. $-6: 15$ p.m.
Time Allowed: 3 Hours

This question paper has $\qquad$ 13 pages (attachments included).

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Instructions to Candidates:This question paper has SIX questions.
    Attempt FIVE questions.
    All questions carry equal marks.
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Attachments: Standard Normal Distribution Table, $t$-distribution Table, $\chi^{2}$-distribution Table, F-distribution Table, Formula Sheets

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1. A social worker of a community centre investigates the weekly total amount of time spent by the members at the centre in the last week. A random sample of members is drawn with results tabulated below:

| Weekly total amount of time (hours) | Number of members |
| :---: | :---: |
| Less than 5 | 3 |
| 5 - less than10 | 5 |
| 10 - less than 15 | 13 |
| 15 - less than 20 | 35 |
| 20 - less than 25 | 19 |
| 25 - less than 30 | 16 |
| 30 - less than 35 | 9 |

(a) Calculate the mean, mode and standard deviation of weekly total amount of time spent at the centre.
(7 marks)
(b) Estimate, from the frequency distribution table, the minimum weekly total amount of time spent at the centre in the last week among the highest $15 \%$ of members.
(3 marks)
(c) Fifty members of the community centre are selected at random. Estimate, from the frequency distribution table, the number of them spent at most 24 hours at the centre in the last week.
(4 marks)
(d) Construct a $95 \%$ confidence interval for the mean weekly total amount of time spent at the centre by all the members in the last week. State your assumption(s) and/or approximation(s).
(6 marks)
2. (a) There are seven shortlisted candidates competing for seven scholarships with distinct amounts. Suppose the chairman of the selection panel prefers two specific candidates to have the top two scholarships and the remaining scholarships are awarded randomly to the other candidates. Calculate the number of ways of matching scholarships with candidates.
(4 marks)
(b) In a recent performance evaluation exercise for the middle-management staff of a company, $65 \%$ of the staff are rated 'Very Satisfactory' by their supervisors and $70 \%$ of the staff are rated 'Very Satisfactory' by their sub-ordinates. Among the staff who are rated 'Very Satisfactory' by their supervisors, $60 \%$ of them are rated 'Very Satisfactory' by their sub-ordinates.
(i) What is the probability that a randomly selected middle-management staff member who is rated 'Very Satisfactory' by his/her supervisor or rated 'Very Satisfactory' by his/her sub-ordinates?
(4 marks)
(ii) It is known that a randomly selected staff of middle-management is not rated 'Very Satisfactory' by his/her supervisor, calculate the probability that the selected staff is rated 'Very Satisfactory' by his/her sub-ordinates. (4 marks)
(Question 2 Cont'd)
(c) A research centre has three senior researchers namely, A, B and C, and they share the projects conducted by the centre according to their research interests. For a project conducted by the centre, the probability that the project relates to the research interest of A, B and C are $0.3,0.38$ and 0.32 , respectively, while their probabilities of generating a revenue above $\$ 200,000$ from a project are $0.3,0.5$ and 0.4 , respectively.
(i) A new project conducted by the centre has generated revenue above $\$ 200,000$, calculate the probability that the project relates to the research interest of A.
(4 marks)
(ii) Five projects are chosen at random and it is found that all of them have generated revenue above $\$ 200,000$, calculate the probability that at least three of them relate to the research interest of A .
(4 marks)
3. (a) Food delivery time of a food delivery company follows the normal distribution approximately with a mean of 30 minutes and a standard deviation of 8 minutes.
(i) Calculate the probability that the delivery time of a randomly selected food delivery job is between 24 minutes and 34 minutes.
(3 marks)
(ii) Determine the maximum delivery time of the fastest $2.5 \%$ of the food delivery jobs. (3 marks)
(iii) Calculate the probability that in 200 randomly selected delivery jobs, at least 110 of them have delivery time within 32 minutes.
(6 marks)
(iv) Calculate the probability that the mean delivery time of 100 randomly selected delivery jobs is between 28 minutes and 33 minutes. ( 4 marks)
(b) From records, among the patients who have completed the treatment of a certain psychological illness, $0.8 \%$ of them suffered from the illness again within 1 year. Calculate the probability that among 1000 randomly selected patients who have completed the treatment, five of them suffered from the illness again within 1 year.
(4 marks)
4. (a) In order to collect the views from employees of a company about a change of dental benefits, a random sample of 250 employees is taken and 185 of them support the change. Construct a $95 \%$ confidence interval for the proportion of all employees who support the change. Interpret your results.
(6 marks)
(b) The sales manager of a chain store studies the difference between the mean daily sales of two new stores. It is known that the population standard deviations of daily sales of the two new stores are $\$ 500$ and $\$ 480$. If he wants to produce a point estimate of the difference between the two population means with error not more than $\$ 100$ with $95 \%$ confidence, determine the required sample size when sample sizes are identical.
(6 marks)

## (Question 4 Cont'd)

(c) In comparing the traffic flow during weekdays and weekends at a certain road junction, random samples of 15 -minute periods are drawn and the summaries of number of vehicles passing through the junction are given in the following table:

|  | Sample size | Mean | Standard deviation |
| :---: | :---: | :---: | :---: |
| Weekdays | 22 | 40 | 6 |
| Weekends | 20 | 32 | 7 |

Can we conclude that the mean traffic flow during weekdays is higher than that during weekends at $5 \%$ level of significance? State your assumptions. (8 marks)
5. (a) In an annual contract renewal exercise of two caterers, namely A and B, who provide catering services for a company, the performance of the caterers are assessed using a number of performance indicators and the scores rated by randomly selected customers are listed below:

| Customer | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | 88 | 76 | 82 | 65 | 70 | 74 | 78 | 85 | 81 |
| B | 80 | 78 | 85 | 60 | 62 | 70 | 73 | 80 | 77 |

Can we conclude that the mean score of caterer $A$ is higher than that of caterer $B$ at $2.5 \%$ level of significance?
(7 marks)
(b) There are special edition products produced for celebrating the $10^{\text {th }}$ anniversary of a company. The special edition products and ordinary products are mixed that five randomly selected products are packed into a box for shipment. A random sample of boxes is selected from a large shipment and their number of special edition products per box are listed below:

| Number of special edition <br> products | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of boxes | 45 | 82 | 50 | 16 | 4 | 1 |

Test whether the number of special edition products in a box follows the Binomial distribution at $5 \%$ level of significance.
(6 marks)
(c) In studying the association of students' preference for a proposed study tour and their level of study, a random sample of students is selected with results tabulated in the following two-way classification table.

| Level of study | Students' preference |  |  |
| :---: | :---: | :---: | :---: |
|  | Not preferred | Neutral | Preferred |
| Freshmen | 15 | 20 | 40 |
| Sophomores | 10 | 15 | 21 |
| Seniors | 14 | 31 | 34 |

Test whether students’ preference for the proposed study tour and their level of study are independent at $2.5 \%$ level of significance.
(7 marks)
6. (a) It is suggested that the academic performance of students (GPA, y) can be affected by the time spent on part-time job per week ( x , in hours). A random sample of eight students is chosen and the summaries of data collected are given below:

$$
\sum x=75, \sum x^{2}=1051, \sum y=27.77, \sum y^{2}=96.6477 \text { and } \sum x y=253.62
$$

(i) Find the least squares linear regression equation for predicting GPA using time spent on part-time job per week.
(ii) Calculate the coefficient of correlation and interpret the result.
(b) In studying the daily traffic flow on a certain highway, an analyst proposes to model the traffic flow on rainfall forecast for the day together with three more independent variables. A random sample of 65 daily data is used to construct the model and part of the output is provided below:

|  | Coefficients | Standard Error |
| :--- | ---: | ---: |
| Intercept | 779.0 | 141.8 |
| Rainfall forecast | 0.1039 | 0.0136 |
| $x_{2}$ | -0.00000222 | 0.000000324 |
| $x_{3}$ | 98.0 | 75.6 |
| $x_{4}$ | 0.00291 | 0.00332 |

In addition, the value of F-statistic is 520.
(i) Find the fitted multiple linear regression equation.
(ii) Test whether rainfall forecast is a significant independent variable in the proposed model at $1 \%$ level of significance.
(iii) Test whether the overall model is significant at $5 \%$ level of significance.
(3 marks)
(iv) Calculate the coefficient of determination and interpret your result. (3 marks)

