The Hong Kong Polytechnic University

AMA1501
Solution to Assignment

1. $\sum f = 60, \sum fx = 2040, \sum fx^2 = 71575$

   (a) $\bar{x} = \frac{\sum x}{\sum f} = \frac{2040}{60} = 34$(million dollars)
   Median $= 30 + \frac{30-15}{24}(35-30) = 33.125$(million dollars)
   $s = \sqrt{\frac{60(71575)-2040^2}{60(59)}} = 6.127183$(million dollars)

   (b) Coefficient of Variation $= \frac{6.127183}{34} \times 100\% = 18.02\%$
   The variability of datasets can be more satisfactorily compared by the coefficient of variation when they have different orders of magnitude or they have different units of measurements.

   (c) $\frac{35-33.5}{35-30}(24) + 12 + \frac{43.5-40}{45-40}(4) = 22$

   (d) $D_8 = 35 + \frac{60(0.8)-39}{12}(40-35) = 38.75$(million dollars)

2. (a) i. Pr(at least one rotten egg) = 1-Pr(no rotten egg is chosen)

$$= 1 - \frac{\binom{3}{0}\binom{7}{3}}{\binom{10}{3}} = 0.7083$$

ii. Pr(2 rotten eggs are chosen—at least one rotten egg)

$$= \frac{\text{Pr(2 rotten eggs are chosen)}}{\text{Pr(at least one rotten egg is chosen)}}$$

$$= \frac{\binom{3}{2}\binom{7}{1}/\binom{10}{3}}{1-\binom{3}{0}\binom{7}{3}/\binom{10}{3}} = \frac{21}{85}$$

(b) Let A be the event that flight attendants will strike

Let B be the event that pilots will strike

$Pr(A) = 0.65, Pr(B) = 0.25, Pr(A|B) = 0.8$

i. $Pr(A \cap B) = Pr(B)Pr(A|B) = 0.25(0.8) = 0.2$

ii. $Pr(\bar{A} \cap \bar{B}) = 1 - Pr(A \cup B) = 1 - (Pr(A) + Pr(B) - Pr(A \cap B)) = 1 - (0.65 + 0.25 - 0.2) = 0.3$

iii. $Pr(\bar{B}|A) = 1 - Pr(B|A) = 1 - \frac{Pr(A \cap B)}{Pr(A)} = 1 - \frac{0.2}{0.65} = \frac{9}{13}$

(c) Let A be the event that a students goes to school by bus.

Let B be the event that a students goes to school by train.

Let C be the event that a students goes to school by light bus.

Let D be the event that a students is late to school.

$P(A) = 0.4, P(B) = 0.2, P(C) = 0.4$

$P(D|A) = 0.15, P(D|B) = 0.35, P(D|C) = 0.1$

$$\begin{aligned} P(B|D) &= \frac{P(B \cap D)}{P(D)} \\ &= \frac{P(B)P(D|B)}{P(A)P(D|A) + P(B)P(D|B) + P(C)P(D|C)} \\ &= \frac{0.2(0.35)}{0.4(0.15) + 0.2(0.35) + 0.4(0.1)} = \frac{7}{17} \end{aligned}$$

3. (a) i. Let X be the amount of annual travel expense (\$)

$X \sim N(15000, 2500^2)$

$P(13000 < X < 16500) = P(-0.8 < Z < 0.6) = 1 - 0.2119 - 0.2743 = 0.5138$

ii. $P(X > a) = 0.1$

$P\left(Z > \frac{a-15000}{2500}\right) = 0.1$

$P(Z > 1.28) = 0.1$

Thus, $a \cong 15000 + 1.28(2500) = \$18200$

iii. $P(X > 13000) = P(Z > -0.8) = 1 - 0.2119 = 0.7881$

Let Y be the number of selected executives having annual travel expenses of over \$13000, out of 250 executives.

$Y \sim Bin(250, 0.7881)$

Since $n > 30, 0.1 < p < 0.9, np > 5, n(1-p) > 5$, therefore we may use the normal approximation to Binomial distribution.

$Y \sim N(250(0.7881) = 197.025, 250(0.7881)(0.2119) = 41.7495975)$

$P(Y > 200) \cong P\left(Z > \frac{200.5-197.025}{\sqrt{41.7495975}}\right) \cong P(Z > 0.54) \cong 0.2946$

(b) i. Let X be the number of claims handled by the agent in a day

$X \sim Poisson(5)$

$P(X < 4) = \sum_{x=0}^{3} \frac{e^{-5}5^x}{x!} = 0.2650$

ii. Let Y be the number of days out of 20 days, having more than 3 claims in a day.

$Y \sim Binomial(20, 1 - 0.2650 = 0.735)$

$P(Y \geq 17) = \sum_{y=18} 20(0.735)^y(0.265)^{20-y} = 0.06967$

4. (a) Let X be the amount of time spent on internet games (hours/day)
$\bar{X} \sim N(3, 1.25^2/36)$
$P(\bar{X} > 3.5) = P\left(Z > \frac{3.5-3}{1.25/\sqrt{36}}\right) = P(Z > 2.4) = 0.0082$

(b) Let X be the monthly expenses of university students in Hong Kong in the last month (\$).
Assumption: $X \sim N(\mu, \sigma^2)$ and random sample of size 10 is selected with replacement or the size of population is infinite.
$\bar{x} = \frac{34891}{10} = 3489.1$
$s = \sqrt{\frac{10(126494667)-34891^2}{10(9)}} = 726.97844$
A 90% confidence interval for μ is
$3489.1 \pm (1.833)\frac{726.97844}{\sqrt{10}}$
Hence, $\$3067.7102 < \mu < \$3910.4898$ Interpretation:
In repeated sampling from the population with the same sample size, and a 90% confidence interval for the population mean is constructed for each sample, it is expected that 90 out of 100 such confidence intervals will contain the true population mean. We do not know whether the constructed confidence interval is one of the 90, but there is a good chance that it is one of them. Hence, we usually state that we have 90% confidence that the true population mean is within the interval ($\$3067.7102 < \mu < \$3910.4898$)

(c) $(1.96)\frac{2.5}{\sqrt{n}} < 0.25$, hence $n > 384.16$. The sample size required is 385.

(d) Assumption: random sample is selected with replacement or population size is inifinitely large.
Approximation: by central limit theorem, sampling distribution of sample proportion follows the normal distribution approximately.
Let p be the population proportion of car owners in Hong Kong purchasing only third party insurance for their vehicles.
A 99% confidence interval for p is
$0.4 \pm 2.576\sqrt{\frac{0.4(0.6)}{500}}$
Hence, $0.3436 < p < 0.4564$