1. (a)

Class marks, x	f	fx	fx^2
215	2	430	92450
225	5	1125	253125
235	4	940	220900
245	8	1960	480200
255	10	2550	650250
265	5	1325	351125
275	4	1100	302500
285	2	570	162450
	40	10000	2513000

Mean,

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{10000}{40} = \$250$$

Standard deviation,

$$S = \sqrt{\frac{n(\sum fx^2) - (\sum fx)^2}{n(n-1)}}$$

$$= \sqrt{\frac{40(251300) - (10,000)^2}{40(39)}}$$

$$= \$18.26$$

(b) $$\text{Median} = 250 + \frac{20-19}{10} \times 10$$

$$= \$251$$

(c) Mean = \$250 Standard deviation = \$18.26

Median = \$251

Pearson's 2nd coefficient of skewness

$$= \frac{3(\text{mean} - \text{median})}{\text{Std. deviation}} = \frac{3(250-251)}{18.26} = -0.164$$

$\therefore$ the distribution is nearly symmetrical.

(d) Percentage of workers whose daily income is at least \$242 but less than \$280

$$= \frac{8 \times \frac{8}{10} + 10 + 5 + 4}{40} \times 100\%$$

$$= \frac{25.4}{40} \times 100\% = 63.5\%$$

(e) $Coefficient\ of\ Variation, CV = \frac{S}{\bar{x}} \times 100\%$

$$CV(QMB) = \frac{18.26}{250} \times 100\% = 7.3\%$$

$$CV(rival\ firm) = \frac{400}{7000} \times 100\% = 5.7\%$$

$\because CV(QMB) > CV(rival\ \ firm)$

Therefore, QMB workers' income is more variable.

2. (a) Frequency Table:

	f	x
-5 - under 0	2	–2.5
0 - under 5	2	2.5
5 - under 10	4	7.5
10 - under 15	8	12.5
15 - under 20	11	17.5
20 - under 25	13	22.5
25 - under 30	6	27.5
30 - under 35	4	32.5
Total	50	

Cumulative Frequency Table:

Under	*Cumulative frequency*
-5	0
0	2
5	4
10	8
15	16
20	27
25	40
30	46
35	50

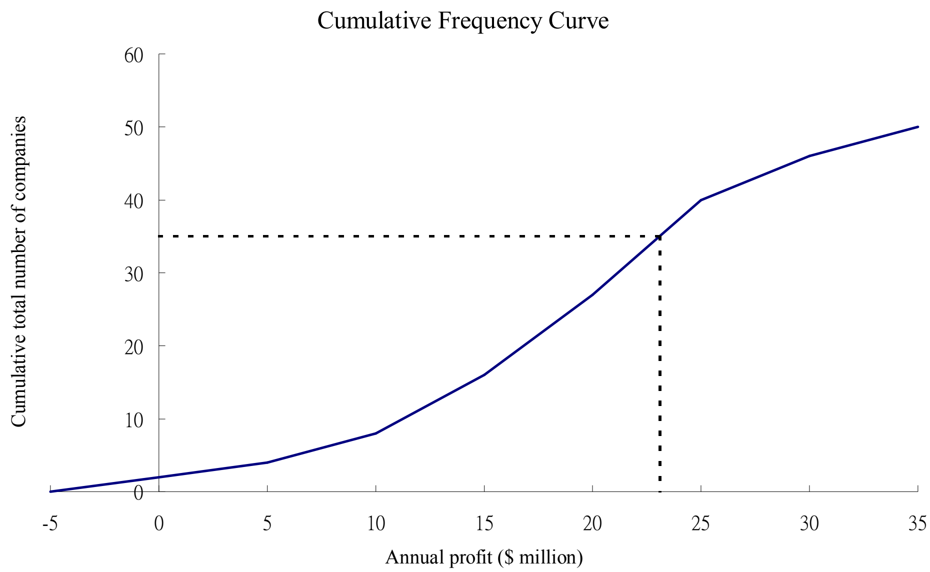
$\sum fx = 910 \quad \sum fx^2 = 20,212.5$

$$\text{mean} = \frac{910}{50} = 18.20 \text{ (\$ million)}$$

$$\text{median} = 15 + \frac{(50/2) - 16}{11} \times 5$$
$$= 19.09 \text{ (\$ million)}$$

$$\text{standard deviation} = \sqrt{\left(\frac{20.212.5}{49} - \frac{910^2}{50 \times 49}\right)}$$
$$= 8.63 \text{ (\$ million)}$$

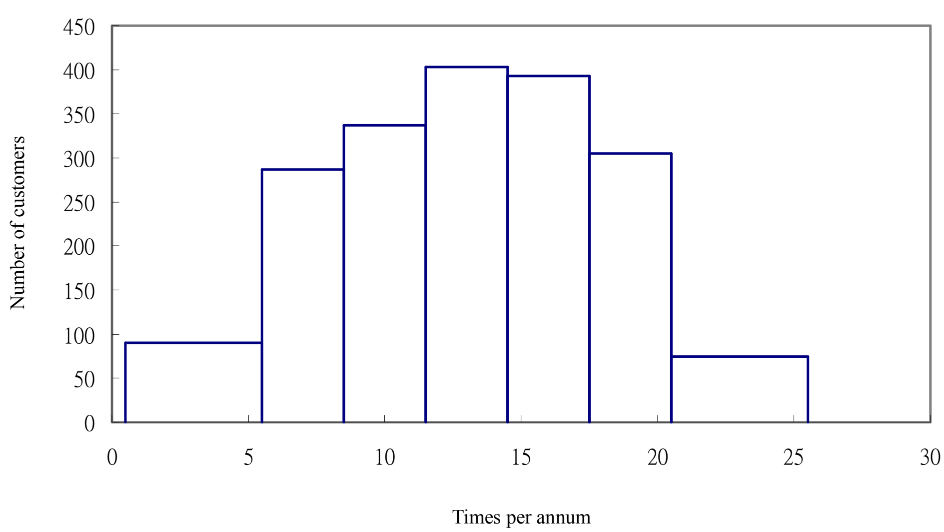
(b) Cumulative frequency curve



(c) Profit exceeded by 30% of the companies is equivalent to profit that is less than by 70% of the companies

= 23.1 ($ million)

3. (a) Histogram



(b) $\sum fx = 26026, \sum fx^2 = 394058, \sum f = 2000$

$$\bar{X} = \frac{\sum fx}{\sum f} = \frac{26026}{2000} = 13.013$$

$$\text{Mode} = 11.5 + \frac{403-337}{(403-337)+(403-393)}(3) = 11.5 + 2.6053 = 14.1053$$

$$\text{Median} = 11.5 + \frac{1000-774}{403}(3) = 11.5 + 1.6824 = 13.1824$$

$$Q_1 = 8.5 + \frac{500-437}{337}(3) = 8.5 + .5608 = 9.0608$$

$$Q_3 = 14.5 + \frac{1500-1177}{393}(3) = 15.5 + 2.4656 = 16.9656$$

$$S = \left[\frac{394058}{1999} - \frac{26026^2}{2000(1999)}\right]^{\frac{1}{2}}$$
$$= [27.7047]^{\frac{1}{2}} = 5.2635$$

$$SK_1 = \frac{\text{Mean} - \text{Mode}}{S} = \frac{13.013 - 14.1053}{5.2635} = -0.2075$$
$$\text{or } SK_2 = \frac{3(\text{Mean} - \text{Median})}{S} = \frac{3(13.013 - 13.1824)}{5.2635} = -0.0966$$

$$C.V. = \frac{S}{\bar{x}} \times 100\% = \frac{5.2635}{13.013} \times 100\% = 40.4480$$

(c) The distribution of customer purchasing behaviour is slightly skewed to the left.

4. Let A be the event of the award for design

Let B be the event of the award for material

$$\Pr(A) = 0.28, \qquad \Pr(B) = 0.13,$$
$$\Pr(A \cup B) = 0.36$$
$$\begin{aligned}\Pr(A \cap B) &= \Pr(A) + \Pr(B) - \Pr(A \cup B) \\ &= 0.28 + 0.13 - 0.36 \\ &= 0.05\end{aligned}$$

5. Let G be the event of good loan

Let B be the event of bad loan

Let L be the event of long-term loan

Let S be the event of short-term loan

$$\Pr(G) = 0.8, \qquad \Pr(B) = 0.2$$
$$\Pr(L \mid G) = 0.7, \quad \Pr(S \mid G) = 0.3$$
$$\Pr(L \mid B) = 0.2, \quad \Pr(S \mid B) = 0.8$$

$$\begin{aligned}\Pr(G \mid L) &= \frac{\Pr(G) \cdot \Pr(L \mid G)}{\Pr(L)} \\ &= \frac{\Pr(G) \cdot \Pr(L \mid G)}{\Pr(G) \cdot \Pr(L \mid G) + \Pr(B) \cdot \Pr(L \mid B)} \\ &= \frac{0.8 \times 0.7}{0.8 \times 0.7 + 0.2 \times 0.2} \\ &= \frac{56}{60} \\ &= \frac{14}{15} \\ &= 0.933\end{aligned}$$

6. There are only 3 cases for at least one of each course,

i.e.

Management	Accountancy	Marketing
2	1	1
1	2	1
1	1	2

Therefore, the required probability

$$= \frac{\binom{6}{2}\binom{7}{1}\binom{3}{1} + \binom{6}{1}\binom{7}{2}\binom{3}{1} + \binom{6}{1}\binom{7}{1}\binom{3}{2}}{\binom{16}{4}}$$

$= 0.45$

7. (a) Let X be random variable of the sales volume in units, $X \sim N(10000,\ 2000^2)$

$$\begin{aligned}
&\Pr(7000 < X < 13000) \\
&= \Pr\left(\frac{7000-10000}{2000} < Z < \frac{13000-10000}{2000}\right) \\
&= \Pr(-1.5 < Z < 1.5) \\
&= 1 - 2 \times 0.0668 \\
&= 0.8664
\end{aligned}$$

(b) Let y be the sales volume for break even,

$$\begin{aligned}
y(20-16) &= 30000 \\
y &= \frac{30000}{4} \\
&= 7500
\end{aligned}$$

$$\begin{aligned}
&\Pr(\text{at least break even}) \\
&= \Pr(y \ge 7500) \\
&= \Pr\left(Z \ge \frac{7500-10000}{2000}\right) \\
&= \Pr(Z \ge -1.25) \\
&= 1 - 0.1056 \\
&= 0.8944
\end{aligned}$$

8. $P\{\text{defective ball pen is among the 3 chosen for testing}\}$

$$= \frac{{}_{19}C_2 \times {}_1C_1}{{}_{20}C_3} = \frac{171}{1140} = 0.15$$

9. $P\{\text{1 defective ball pen is found among the 3 chosen for testing}\}$

$$= \frac{15}{20} \times \frac{{}_8C_2 \cdot {}_2C_1}{{}_{10}C_3} + \frac{5}{20} \times \frac{{}_9C_2 \cdot {}_1C_1}{{}_{10}C_3}$$

$$= \frac{15}{20} \times \frac{56}{120} + \frac{5}{20} \times \frac{36}{120} = 0.425.$$

10. Let X be the life of the randomly selected motor.

Then $X \sim N(12\ , 3^2)$,

$$\begin{aligned} P(10 < X < 15) &= P\left(\frac{10-12}{3} < \frac{X-\mu}{\sigma} < \frac{15-12}{3}\right) \\ &= P(-0.67 < Z < 1) \\ &= 1 - 0.2514 - 0.1587 \\ &= 0.5899 \end{aligned}$$

11. Let A be the event of the defective part detected by 1st inspector
B be the event of the defective part detected by 2nd inspector

P(A) = 0.85, P(B) = 0.85, P(A $\cap$ B) = 0.8

(a) $$\begin{aligned} P(B \mid A') &= P(B \cap A') / P(A') \\ &= \frac{P(B) - P(A \cap B)}{1 - P(A)} \\ &= \frac{0.85 - 0.80}{1 - 0.85} = 0.333 \end{aligned}$$

(b) $$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.85 + 0.85 - 0.80 \\ &= 0.90 \end{aligned}$$

(c) $$\begin{aligned} 1 - \{P(A \cup B)\}^3 &= 1 - 0.90^3 \\ &= 0.271 \end{aligned}$$

12. Let X be the random variable of the number of customer who turn up in the 55

$X \sim b\ (55, 0.85)$

$$\text{Mean} = \text{np} = 55(0.85) = 46.75$$

$$\sigma = \sqrt{npq} = \sqrt{55(0.85)(0.15)} = 2.6481$$

Required probability:

$$\Pr(X \leq 50)$$

Since $np = 46.75 > 5$, $nq = 55\left(1-0.85\right) = 8.25 > 5$ and $0.1 < p < 0.9$, normal approximation to binomial distribution can be used.

By Normal approximation, the required probability

$$\begin{aligned}\Pr(X < 50.5) &= \Pr\left(Z < \frac{50.5-46.75}{2.6481}\right)\\ &= \Pr(Z < 1.42)\\ &= 1-0.0778\\ &= 0.9222\end{aligned}$$

13. Let the mean of the distribution of the number of errors per page be λ and X be the Poisson random variable. Then

$$\Pr(X = 0) = \frac{e^{-\lambda}\lambda^{\circ}}{0!} = 0.14$$

Hence $\lambda = 1.966$ and

$$\begin{aligned}\Pr(X = 1) &= \frac{e^{-1.966}(1.966)^{1}}{1!}\\ &= 0.275\end{aligned}$$

The required percentage is 27.5%.

14. (a) Let X be the random variable of the number of error per 2 pages.

$\lambda = 4$ error per 2 pages, $X \sim \text{Po}(\lambda)$

$$P(X<3)=\sum_{x=0}^{2}\frac{e^{-4}4^x}{x!}$$
$$=e^{-4}\left\{\frac{4^0}{0!}+\frac{4^1}{1!}+\frac{4^2}{2!}\right\}$$
$$=0.2381$$

(b) $$P(X>5)=1-P(X\le 5)$$
$$=1-\sum_{x=0}^{5}\frac{e^{-4}4^x}{x!}$$
$$=1-e^{-4}\left\{1+4+8+\frac{4^3}{3!}+\frac{4^4}{4!}+\frac{4^5}{5!}\right\}$$
$$=0.2149$$

15. (a) Let H be the event of an employee who was in favour of the modified health care plan

W be the event of an employee who was in favour of the proposal to change the work schedule.

$P(H)=0.42,\quad P(W)=0.22,\quad P(W\mid H)=0.34$

$$P(H\cap W)=P(H)P(W\mid H)$$
$$=0.42\times 0.34=0.1428$$

(b) $$P(H\cup W)=P(H)+P(W)-P(H\cap W)$$
$$=0.42+0.22-0.1428=0.4972$$

(c) $$P(H\mid W)=\frac{P(H\cap W)}{P(W)}=\frac{0.1428}{0.22}=0.6491$$

16. (a) Let X be the rate of return (%)

$$X \sim N(12.2, 7.2^2)$$

$$P(X > 20) = P\left(Z > \frac{20-12.2}{7.2}\right)$$
$$= P(Z > 1.08) = 0.1401$$

(b) $$P(X < 0) = P\left(Z < \frac{0-12.2}{7.2}\right)$$
$$= P(Z < -1.69) = 0.0455.$$

17. $n_1 = n_2 = 2.20, \quad \hat{p}_1 = 0.36, \quad \hat{p}_2 = 0.30$

Let P_1, P_2 be the True proportion of response to single-sheet questionnaire and two-sheet questionnaire respectively.

Then a 95% C.I. for $P_1 - P_2$ is

$$(\hat{p}_1 - \hat{p}_2) \pm 1.96\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$\Rightarrow (0.36-0.30) \pm 1.96\sqrt{\frac{0.36(0.64)}{220} + \frac{0.30(0.70)}{220}}$$

$$\Rightarrow 0.06 \pm 0.088 \Rightarrow (-0.028, 0.148)$$

18. Let P be the population proportion.

$$n = 225, \quad x = 67 \quad \hat{p} = \frac{67}{225} = 0.298$$

$\therefore$ A 95% C.I. estimate for P is

$$\hat{p} \pm 1.96\sqrt{\left(\frac{\hat{p}(1-\hat{p})}{n}\right)}$$

$$\text{i.e.} \quad 0.298 \pm 1.96\sqrt{\left(\frac{0.298(0.702)}{225}\right)}$$

$$\text{i.e.} \quad 0.298 \pm 0.060, \quad \text{i.e.} \quad (0.238, 0.358)$$

19. H_0 : Printing errors are independent of type size.

H_1 : Printing errors are dependent on type size.

$\alpha = 0.05,\ \nu = (2-1)(3-1) = 2$

$\therefore$ CR: $\chi^2 > \chi^2_{0.05} = 5.991$

Computation:

	Type Size			Total
	A	B	C	
Pages with errors	23 (29.9)	17 (22.7)	41 (28.4)	81
Pages without errors	241 (234.1)	183 (177.3)	210 (222.6)	634
Total	264	200	251	715

$$\text{Test statistic, } \chi^2 = \frac{6.9^2}{29.9} + \frac{5.7^2}{22.7} + \frac{12.6^2}{28.4} + \frac{6.9^2}{234.1} + \frac{5.7^2}{177.3} + \frac{12.6^2}{222.6}$$
$$= 9.714$$

Conclusion: Reject H_0.

20. (a) Let μ_1, μ_2 be the true mean absenteeism of recent ex-smokers and long-term ex-smokers, respectively.

$$n_1 = 34 \quad \bar{x}_1 = 2.21 \quad s_1 = 2.21$$
$$n_2 = 68 \quad \bar{x}_2 = 1.47 \quad s_2 = 1.69$$

$H_0 : \mu_1 = \mu_2$, i.e. $\mu_1 - \mu_2 = 0$

$H_1 : \mu_1 \neq \mu_2$

$\alpha = 0.05$

$CR : Z > 1.96 \quad and \quad Z < -1.96$

Test statistic, $z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)}}$

$$= \frac{(2.21 - 1.47) - 0}{\sqrt{\left(\dfrac{2.21^2}{34} + \dfrac{1.69^2}{68}\right)}}$$

$= 1.717$

Conclusion: Do not reject H_0

(b) Level of significance (α) is the probability of committing type I error in a significance test. In general, α is set equal to 0.05. However, if the consequence of committing type I error in a significance test is serious in terms of monetary loss, then a low value, for example, 0.01, should be used instead.

21. (a) $n = 545 \quad x = 117 \quad \hat{p} = \frac{117}{545} = 0.2147$

Let P be the population proportion of accountants finding cash flow the most difficult estimates to derive.

$H_0 : p = 0.25; \quad H_1 : p < 0.25$

$\alpha = 0.05; \quad CR : Z < -1.645$

Test statistic, $z = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{0.2147 - 0.25}{\sqrt{\frac{0.25(0.75)}{545}}}$
$= -1.903$

Conclusion: Reject H_0.

(b) $c = p - 1.645\sqrt{\frac{0.25(0.75)}{545}}$
$= 0.25 - 0.0305$
$= 0.2195$

So probability of rejecting H_0 when the true $P = 0.28$ is

$$P\{\hat{p} < 0.2195\} = P\left\{Z < \frac{0.2195 - 0.28}{\sqrt{\frac{0.28(0.72)}{545}}}\right\}$$
$$= P\{Z < -3.145\}$$
$$= 0.00083.$$

22. (a) $n = 8 \quad \sum x = 192 \quad \sum y = 608$
$\sum x^2 = 4{,}902 \quad \sum xy = 15{,}032 \quad \sum y^2 = 47{,}094$

$b = \frac{n\sum xy - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2} = \frac{8(15{,}032) - 192(608)}{8(4{,}902) - 192^2}$
$= \frac{3{,}520}{2{,}352} = 1.4966$

$a = \overline{y} - b\overline{x} = 76 - 1.4966(24) = 40.0816$

$\therefore \hat{y} = 40.0816 + 1.4966x$

(b) $\hat{y} = 40.0816 + 1.4966(30) = 84.9796$

(c) $r = \dfrac{n\sum xy - (\sum x)(\sum y)}{\sqrt{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}} = \dfrac{3520}{\sqrt{2352(7088)}}$

$= \dfrac{3520}{4083.0107} = 0.8621$

This value of r shows a strong positive relationship between hours of study and examination grade.

(d) $r^2 = (0.8621)^2 = 0.7432$

This means hours of study can explain 74.32% of variation on examination grade in the regression equation.

23. (a)

$$n = 10 \quad \Sigma x = 0.56 \quad \Sigma y = 96.7$$

$$\Sigma x^2 = 0.0367 \quad \Sigma xy = 5.8$$

$$b = \frac{n\Sigma xy - (\Sigma x)(\Sigma y)}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{(10)(5.8) - (0.56)(96.7)}{(10)(0.0367) - 0.56^2} = 72.060$$

$$a = \overline{y} - b\overline{x} = \frac{96.7}{10} - (72.060)(\frac{0.56}{10})$$

$$= 5.635$$

$$\therefore \hat{y} = 5.635 + 72.060x$$

(b) The slope of the linear regression line is 72.060 and this implies that the P/E ratio of a company is expected to be increased by 0.72060 for each 0.01 increase in its R/S ratio, i.e. for each 1 cent increase in research & development expenditure per dollar of sales.

(c) If

$$x = 0.060,$$

then

$$\hat{y} = 5.635 + (72.060)(0.060)$$
$$= 9.96$$

Reliability of this estimate is low because n is equal to 10 only and $r = 0.57$ (i.e. only 0.32% of the variation in y is explained by the regression line).

24. (a) $\hat{ltimp} = -0.02686 + 0.79116 \times Foreimp + 0.60484 \times Midsole$

(b) For a given midsole impact, each increase of one unit in forefoot impact absorbing capability is expected to result in an average increase in the long-term ability to absorb shock by 0.79116 units. For a given forefoot impact absorbing capability, each increase of one unit in midsole impact is expected to result in the average increase in the long-term ability to absorb shock by 0.60484 units.

(c) $R^2 = \dfrac{12.6102}{13.38473} = 0.9421$. 94.21% of the variation in the long-term ability to absorb shock can be explained by variation in forefoot absorbing capability and variation in midsole impact.

25. (a) $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$

H_1 : at least one β_i is not equal to 0

Under H_0, test statistic f = 39.86 with p-value = 0.000

Decision: Reject H_0

Conclusion: The multiple regression equation is significant at 5% level of significance.

(b) Permits: for each additional building permit, the drywall demand is expected to increase by 476.31.
Mortgage rates: for each additional percentage point, the drywall demand is expected to increase by 1699.
Apartment vacancy rate: for each additional percentage point, the drywall demand is expected to decrease by 1052.8.
Office vacancy rate: for each additional percentage point, the drywall demand is expected to increase by 130.8.

(c) To test $H_0 : \beta_i = 0$ against $H_1 : \beta_i \neq 0$ for all the independent variables, the test statistic t of permits, mortgage rates, apartment vacancy rate and office vacancy rate are 12.06, 1.12, -1.65 and 0.47 respectively and the p-values of permits, mortgage rates, apartment vacancy rate and office vacancy rate are 0.000, 0.276, 0.116 and 0.645 respectively. Only one of the independent variables – permits has significant effect on the drywall demand at 5% level of significance.