AMA1501 Introduction to Statistics for Business
Mock Examination Paper 5 Outline Suggested Solution

1. (a)

Class mark (x)	Frequency, f
150	3
250	7
350	16
450	26
600	21
775	13
925	9
1075	5

$\sum f = 100 \qquad \sum fx = 55875 \quad \sum fx^2 = 36576875$

Mean = $\frac{55875}{100} = 558.75$ ($'000)

Standard deviation = $\sqrt{\frac{100(36576875) - 55875^2}{100(100-1)}} = 232.61184$ ($'000)

Mode = $400 + \frac{26-16}{(26-16)+(26-21/2)}(500-400) = 439.21568$ ($'000)

(b) $SK_1 = \frac{558.75 - 439.21568}{232.61184} = 0.5139$

(c)

Monthly revenue less than ($'000)	Cumulative frequency
100	0
200	3
300	10
400	26
500	52
700	73

Monthly revenue less than ($'000)	Cumulative frequency
850	86
1000	95
1150	100

$Q_1 = 300 + \frac{15}{16}(400 - 300) = 393.75$ ($'000)

$Q_3 = 700 + \frac{2}{13}(850 - 700) = 723.07692$ ($'000)

IQR = 723.07692 – 393.75 = 329.32692 ($'000)

(d) $\hat{p} = \left(\frac{400-350}{400-300} \times 16 + 26 + 21 + \frac{780-700}{850-700} \times 13 \right) \Big/ 100 = 0.6193$

A 95% C.I. for p is $0.6193 \pm 1.96\sqrt{\frac{0.6193 \times (1-0.6193)}{100}}$, i.e. 0.5242 < p < 0.7145

2.

(a) (i) No. of panels can be formed = ${}_{30}P_2 \times {}_{28}C_5 = 85503600$

(ii) $\frac{{}_{10}P_2 \times {}_8C_1 \times {}_{20}C_4}{85503600} = 0.0408$

(b) A: highly motivated supervisor

B: hard working supervisor

$\Pr(A) = 0.83 \quad \Pr(B) = 0.9 \quad \Pr(\bar{A} \cap \bar{B}) = 0.05$

(i) $\Pr(A \cap B) = 0.83 + 0.9 - (1 - 0.05) = 0.78$

(ii) $\Pr(\bar{A} | B) = 1 - \frac{0.78}{0.9} = 0.1333$

(c) HR – selected respondent works in HR Dept.

IT – selected respondent works in IT Dept.

GA – selected respondent works in GA Dept.

A – selected respondent satisfied with the working environment

$\Pr(HR) = 0.25 \qquad \Pr(IT) = 0.35 \qquad \Pr(GA) = 0.4$

$\Pr(A|HR) = 0.9 \qquad \Pr(A|IT) = 0.85 \qquad \Pr(A|GA) = 0.88$

$$\Pr(IT|A) = \frac{0.35\times 0.85}{0.25\times 0.9 + 0.35\times 0.85 + 0.4\times 0.88} = 0.3402$$

X – number of respondents who work in IT Dept given that the respondent satisfied with the working environment

$X \sim B(3, 0.3402)$

$$\Pr(X \geq 2) = {}_3C_2 (0.3402)^2 (1-0.3402) + {}_3C_3 (0.3402)^3 = 0.2685$$

3. (a) X – weight of package (grams), $X \sim N(310, 8^2)$

(i) $\Pr(X < 300) = \Pr(Z < -1.25) = 0.1056$

(ii) $\Pr(X < a) = \Pr\left(Z < \frac{a-310}{8}\right) = 0.05 \Rightarrow \frac{a-310}{8} = -1.645 \Rightarrow a = 296.84$ grams

(iii) $\bar{X} \sim N(310, 8^2/20)$

$\Pr(308 < \bar{X} < 314) \approx \Pr(-1.12 < Z < 2.24) = 0.856$

(b) X – number of enquiries that were classified as hardware problem

$X \sim B(60, 0.4)$

Since n>30, np=24>5, nq=36>5 and 0.1<p<0.9, $X \sim N(24, 14.4)$ approximately

$\Pr(X > 30) = \Pr(X > 30.5) \approx \Pr(Z > 1.71) = 0.0436$

(c) X – number of complaints per day

X ~ Po(2)

$$\Pr(X \leq 1) = \sum_{x=0}^{1} \frac{e^{-2} 2^x}{x!} = 0.4060$$

Y – number of days in a month of 31 days which have at most 1 complaint per day

Y~B(31, 0.4060)

Since n>30, np>5, nq>5 and 0.1<p<0.9, $Y \sim N(12.5862, 2.73425^2)$ approximately

$\Pr(Y \geq 16) = \Pr(Y > 15.5) \approx \Pr(Z > 1.07) = 0.1423$

4. (a) $\bar{X}_1 - \bar{X}_2$ - difference of average weekly sales ($)

$$\bar{X}_1 - \bar{X}_2 \sim N\left(100000 - 100000, \frac{20000^2}{4} + \frac{20000^2}{4}\right)$$

$$\Pr\left(\bar{X}_1 - \bar{X}_2 < -10000\right) + \Pr\left(\bar{X}_1 - \bar{X}_2 > 10000\right) \approx \Pr\left(Z < -0.71\right) + \Pr\left(Z > 0.71\right) = 0.4778$$

(b) A 95% confidence interval for mean time spent per week is

$1.5 \pm 2.064 \times \frac{0.8}{\sqrt{25}}$, i.e., $1.1698 < \mu < 1.8302$

(c) X – weight of cake (grams)

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$H_0: \ \mu = 100$

$H_1: \ \mu < 100$

$\alpha = 0.01$

Critical region: z < - 2.33

Under H_0, test statistic $z = \frac{94.6 - 100}{2.5/\sqrt{36}} = -12.96$

Decision: Reject H_0

(d)

$\bar{X}_1$ - sample mean score rated by senior staff

$\bar{X}_2$ - sample mean score rated by junior staff

$\bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)$ approximately

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 < \mu_2$

$\alpha = 0.01$

Critical region: z < -2.33

Under H_0, test statistic $z = \frac{(78 - 81) - 0}{\sqrt{\frac{9^2}{40} + \frac{8^2}{60}}} = -1.706$

Decision: Do not reject H_0

5. (a) D – paired difference of score

d: 6 11 6 6 9 5 9 0 -1 7

n=10, $\sum d = 58$, $\sum d^2 = 466$

$\bar{d} = \frac{58}{10} = 5.8$, $s_d = \sqrt{\frac{10(466) - 58^2}{10(10-1)}} = 3.7947$

$H_0: \ \mu_d = 0$

$H_1: \ \mu_d > 0$

$\alpha = 0.05$

Critical region: t > 1.833, $\nu = 9$

Under H_0, test statistic $t = \frac{5.8 - 0}{3.7947/\sqrt{10}} = 4.8333$

Decision: Reject H_0

(b)

$H_0:$ there is no preference between the models

$H_1:$ there is preference between the models

$\alpha = 0.05$

$E_i = 125, i = 1, \cdots, 4$

Critical region: $\chi^2 > 7.815, \nu = 3$

Under H_0, test statistic $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 9.2$

Decision: Reject H_0

(c) $H_0:$ favouring the new design or not is independent of the age

$H_1:$ favouring the new design or not and the age of customer are dependent

$\alpha = 0.01$

Critical region: $\chi^2 > 9.210, \nu = 2$

Expected frequencies:

	Teenager	Adult	Elderly
Favour the new design	120.05	120.62	69.33
Do not favour the new design	92.95	93.38	53.67

Under H_0, test statistic $\chi^2 = \sum \frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}} = 32.47$

Decision: Reject H_0

6. (a) (i)

$$b = \frac{10(56089) - (683)(813)}{10(47405) - (683)^2} = 0.7421$$

$$a = \frac{813}{10} - 0.7421 \times \frac{683}{10} = 30.6147$$

$$\hat{y} = 30.6147 + 0.7421x$$

(ii) When x = 80, $\hat{y} = 30.6147 + 0.7421 \times 80 = 89.9825 \approx 90$

(b) (i) $\hat{\text{Sales}} = -29.6972 + 0.3209(\text{Population size}) + 4.8782(\text{Advertising cost})$

(ii)

$a = 38798.92 - 2094.42 = 36704.5$

$b = 2$

$c = 12 - 2 - 1 = 9$

$d = 12 - 1 = 11$

$e = 36704.5/2 = 18352.25$

$f = 2094.42/9 = 232.713$

$g = 18352.25/232.713 = 78.86$

(iii) $R^2 = \frac{36704.5}{38798.92} = 0.946$

(iv) $H_0 : \beta_1 = 0$

$H_1 : \beta_1 \neq 0$

$\alpha = 0.05$

Critical region: t < -2.262 and t > 2.262, $\nu = 9$

Under H_0, test statistic $t = \dfrac{0.3209 - 0}{0.0484} = 6.63$

Decision: Reject H_0