AMA1501 Introduction to Statistics for Business
Mock Examination Paper 6 Outline Suggested Solution

1. (a)

Class mark (x)	Frequency, f
100	7
300	12
500	19
700	26
900	18
1250	10
1750	5
2500	3

$\sum f = 100 \qquad \sum fx = 76950 \quad \sum fx^2 = 82907500$

Mean = $\dfrac{76950}{100} = \$769.50$

Standard deviation = $\sqrt{\dfrac{100(82907500) - 76950^2}{100(100-1)}} = \$489.22$

Mode = $600 + \dfrac{26-19}{(26-19)+(26-18)}(800-600) = \$693\dfrac{1}{3}$

(b)

Amount less than ($'000)	Cumulative frequency
0	0
200	7
400	19
600	38
800	64
1000	82
1500	92
2000	97
3000	100

$$D_9 = 1000 + \frac{90-82}{10}(1500-1000) = \$1400$$

(c) Let y be the amount of reimbursement

y: 100 300 500 700 900 1000 1000 1000

$\sum fy = 66200$ $\sum fy^2 = 51220000$

Mean = $\frac{66200}{100} = \$662$

Standard deviation = $\sqrt{\frac{100(51220000)-66200^2}{100(100-1)}} = \$273.32$

(d) $\hat{p} = \left(\frac{1000-900}{1000-800} \times 18 + 10 + 5 + 3\right) \Big/ 100 = 0.27$

Let X be the number of receipts have the amount greater than \$900

X ~ B(5, 0.27)

$$\Pr(X \le 2) = \sum_{x=0}^{2} {}_5C_x (0.27)^x (0.73)^{5-x} = 0.8743$$

2.

(a) (i) Pr('Job satisfaction' is not the most important factor)

= 1- Pr('Job satisfaction' is the most important factor)

= $1 - \frac{6!}{7!} = \frac{6}{7}$

(ii) Pr('Job satisfaction' is most important and 'Working environment' is least important)

= $\frac{1!5!1!}{7!} = \frac{1}{42}$

(b) A: graduate is a Bachelor

B: graduate is employed

$\Pr(A) = 0.8$ $\Pr(B) = 0.4$ $\Pr(B|A) = 0.3$

(i) $\Pr(\bar{A} \cap \bar{B}) = 1 - [0.8 + 0.4 - 0.8 \times 0.3] = 0.04$

(ii) $\Pr(A|\bar{B}) = \frac{\Pr(A) - \Pr(A \cap B)}{1 - \Pr(B)} = \frac{0.8 - 0.8 \times 0.3}{1 - 0.4} = \frac{14}{15}$

(c) A – a batch of bottle is supplied by Factory A

B – a batch of bottle is supplied by Factory B

C – a batch of bottle is supplied by Factory C

D – a batch has one broken bottle

$\Pr(A) = 0.55 \quad \Pr(B) = 0.3 \quad \Pr(C) = 0.15$

$\Pr(D|A) = {}_{10}C_1 (0.05)^1 (0.95)^9 = 0.3151$

$\Pr(D|B) = {}_{10}C_1 (0.08)^1 (0.92)^9 = 0.3777$

$\Pr(D|C) = {}_{10}C_1 (0.1)^1 (0.9)^9 = 0.3874$

$$\Pr(A|D) = \frac{0.55 \times 0.3151}{0.55 \times 0.3151 + 0.3 \times 0.3777 + 0.15 \times 0.3874} = 0.5027$$

3. (a) X – delivery time (minutes), $X \sim N(45, 8^2)$

(i) $\Pr(X \leq 43) = \Pr(Z \leq -0.25) = 0.4013$

(ii) Let k by the required maximum delivery time

$$\Pr(X < k) = \Pr\left(Z < \frac{k-45}{8}\right) = 0.9 \Rightarrow \frac{k-45}{8} = 1.282 \Rightarrow k = 55.25 \text{ minutes}$$

(iii) $\bar{X} \sim N(45, 8^2/10)$

$\Pr(40 < \bar{X} < 47) \approx \Pr(-1.98 < Z < 0.79) = 1 - 0.0239 - 0.2148 = 0.7613$

(b) X – number of students purchase the textbook

X ~ B(180,0.8)

Since n>30, np>5, nq>5 and 0.1<p<0.9, normal approximation is used.

$\mu = 180 \times 0.8 = 144, \sigma^2 = 180 \times 0.8 \times 0.2 = 28.8$

$\Pr(X \geq 140) = \Pr(X > 139.5) \approx \Pr(Z > -0.84) = 1 - 0.2005 = 0.7995$

(c) X – number of products sold in 1 hour

$X \sim Po(6)$

$$\Pr(5 \leq X \leq 7) = \sum_{x=5}^{7} \frac{e^{-6} 6^x}{x!} = 0.16062 + 0.16062 + 0.13768 = 0.45892$$

Y – number of hours having hourly sales of 5 – 7 products

Y~B(10, 0.45892)

$$\Pr(X=5) = {}_{10}C_5 (0.45892)^5 (1-0.45892)^5 = 0.2379$$

4. (a) $\bar{x} = \frac{21.4}{10} = 2.14$ minutes, $s = \sqrt{\frac{10(49.08)-21.4^2}{10(10-1)}} = 0.60406$ minutes

A 95% confidence interval for mean completion time is

$2.14 \pm 2.262 \times \frac{0.60406}{\sqrt{10}}$, i.e., $1.7079 < \mu < 2.5721$ (minutes)

(b) $H_0: p_1 = p_2$

$H_1: p_1 < p_2$

$\alpha = 0.025$

Critical region: z < -1.96

$\hat{p}_1 = 0.48$ $\qquad$ $\hat{p}_2 = 0.72$ $\qquad$ $\hat{p} = \frac{240+360}{500+500} = 0.6$

Under H_0, test statistic $z = \frac{(0.48-0.72)-0}{\sqrt{0.6(0.4)\left(\frac{1}{500}+\frac{1}{500}\right)}} = -7.746$

Decision: Reject H_0

(c) D = price quoted by supplier A – price quoted by supplier B

d: 100 -5 0 50 100 50 25 -20 10 100

$\bar{d} = \frac{410}{10} = \$41$, $s = \sqrt{\frac{10(36150)-410^2}{10(10-1)}} = \$46.3561$

$H_0: \mu_D = 0$

$H_1: \mu_D > 0$

$\alpha = 0.05$

Critical region: t > 1.833

Under H_0, test statistic $t = \frac{41-0}{46.3561/\sqrt{10}} = 2.7969$

Decision: Reject H_0

5. (a)

$H_0 : \mu = 8000$

$H_1 : \mu < 8000$

$\alpha = 0.01$

Critical region: t < -2.821

Under H_0, test statistic $t = \dfrac{6500 - 8000}{1200/\sqrt{10}} = -3.953$

Decision: Reject H_0

(b)

$H_0 :$ number of complaints received in an hour follows Poisson distribution

$H_1 :$ H_0 is false

$\alpha = 0.05$

$\bar{x} = \dfrac{103}{100} = 1.03$

<table>
<tr><td>No. of complaints</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>≥ 5</td></tr>
<tr><td>O_i</td><td>33</td><td>40</td><td>19</td><td>7</td><td>1</td><td>0</td></tr>
<tr><td>E_i</td><td>35.7</td><td>36.77</td><td>18.94</td><td>6.50</td><td>1.67</td><td>0.41</td></tr>
<tr><td>E_i</td><td>35.7</td><td>36.77</td><td>18.94</td><td colspan="3">8.59</td></tr>
</table>

Critical region: $\chi^2 > 5.991, \nu = 2$

Under H_0, test statistic $\chi^2 = \sum \dfrac{(O_i - E_i)^2}{E_i} = 0.5285$

Decision: Do not reject H_0

(c) $H_0 :$ air quality and temperature are independent

$H_1 :$ air quality and temperature are not independent

$\alpha = 0.01$

Critical region: $\chi^2 > 13.277, \nu = 4$

Expected frequencies:

	Air quality		
Temperature	Poor	Fair	Good
Below average	3.5	6.3	18.2
Average	14.5	26.1	75.4
Above average	7.0	12.6	36.4

Under H_0, test statistic $\chi^2 = \sum \frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}} = 10.789$

Decision: Do not reject H_0

6. (a) (i)

$$b = \frac{8(31731.7) - (4644)(46.2)}{8(3208996) - (4644)^2} = 0.009573$$

$$a = \frac{46.2}{8} - 0.009573 \times \frac{4644}{8} = 0.217674$$

$$\hat{y} = 0.217674 + 0.009573x$$

(ii) When x = 700, $\hat{y} = 0.217674 + 0.009573 \times 700 = 6.919015 \times 10^8$ Btu

(b) (i) $\widehat{\text{Sales}} = 34.1046 + 3.7459(\text{Budget}) - 30.0463(\text{Ratio}) + 0.0859(\text{Income})$

(ii)

$a = 29657.75375 - 28741.07878 = 916.67497$

$b = 3$

$c = 14 - 3 - 1 = 10$

$d = 14 - 1 = 13$

$e = 28741.07878/3 = 9580.36$

$f = 916.67497/10 = 91.6675$

$g = 9580.36/91.6675 = 104.51$

(iii) $1 - R^2 = 1 - \frac{28741.07878}{29657.75375} = 0.0309$

(iv) $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$

H_1 : at least one $\beta_i \neq 0, i = 1, 2, 3$

$\alpha = 0.05$

Critical region: f > 3.71

Under H_0, test statistic f = 104.51

Decision: Reject H_0