AMA1501 Introduction to Statistics for Business
Mock Examination Paper 7 Outline Suggested Solution

1. (a)

Class mark (x)	Frequency, f
13.95	2
15.95	6
17.95	11
19.95	17
21.95	28
23.95	21
26.45	9
29.45	6

$\sum f = 100$ $\sum fx = 2192.5$ $\sum fx^2 = 49262.25$

Mean = $\dfrac{2192.5}{100} = 21.925 \ kg/m^2$

Standard deviation = $\sqrt{\dfrac{100(49262.25) - 2192.5^2}{100(100-1)}} = 3.4695 \ kg/m^2$

Mode = $20.95 + \dfrac{28-17}{(28-17)+(28-21)}(22.95-20.95) = 22.1722 \ kg/m^2$

(b)

BMI less than	Cumulative frequency
12.95	0
14.95	2
16.95	8
18.95	19
20.95	36
22.95	64
24.95	85
27.95	94
30.95	100

$$Q_1 = 18.95 + \frac{25-19}{17}(20.95-18.95) = 19.6559 \ \ kg/m^2$$

$$Q_3 = 22.95 + \frac{75-64}{21}(24.95-22.95) = 23.9976 \ \ kg/m^2$$

IQR = 23.9976 – 19.6559 = 4.3417 kg/m^2

(c) $CV_{male} = \frac{3.4695}{21.925} \times 100\% = 15.82\%$

$CV_{female} = \frac{3.3}{20.3} \times 100\% = 16.26\%$

(d) A 95% C.I. for μ is $21.925 \pm 1.96\sqrt{\frac{3.4695^2}{100}}$, i.e. $21.2450 < \mu < 22.6050 \ \ kg/m^2$

2.

(a) Pr(at least one of programmes A and B would be selected)

$= 1 - \frac{{}_{18}C_5 \times {}_2C_0}{{}_{20}C_5} = \frac{17}{38}$

(b) A: pass theoretical test

B: pass practical test

$\Pr(A) = 0.55 \quad \Pr(B) = 0.4 \quad \Pr(\bar{A} \cap \bar{B}) = 0.4$

(i) $\Pr(A \cap B) = 0.55 + 0.4 - (1 - 0.4) = 0.35$

(iii) $\Pr(\bar{B} | A) = 1 - \frac{0.35}{0.55} = \frac{4}{11}$

(c) A – china plate is supplied by Supplier A

B – china plate is supplied by Supplier B

C – china plate is supplied by Supplier C

D – 2 surface imperfections are found

$\Pr(A) = 0.5 \quad \Pr(B) = 0.2 \quad \Pr(C) = 0.3$

$$\Pr(D|A) = \frac{e^{-2.5}(2.5)^2}{2!} = 0.2565$$

$$\Pr(D|B)=\frac{e^{-3}(3)^2}{2!}=0.2240$$

$$\Pr(D|C)=\frac{e^{-2.8}(2.8)^2}{2!}=0.2384$$

$$\Pr(A|D)=\frac{0.5\times0.2565}{0.5\times0.2565+0.2\times0.2240+0.3\times0.2384}=0.5244$$

3. (a) X – sales in the last month (\$), $X\sim N\left(100000,25000^2\right)$ approximately

(i) $\Pr(80000<X<130000)=\Pr(-0.8<Z<1.2)=0.6730$

(ii) $\Pr(X<a)=\Pr\left(Z<\frac{a-100000}{25000}\right)=0.05\Rightarrow\frac{a-100000}{25000}=-1.645\Rightarrow a=58875$

(iii)

$$\Pr\left(\min_{1\le i\le 5}X_i\ge110000\right)=\prod_{i=1}^{5}\Pr(X_i\ge110000)=\left[\Pr(Z\ge0.4)\right]^5=0.004859$$

(b) X – number of employees who drive to work

$X\sim B(25,0.8)$

$$\Pr(X\le20)=1-\sum_{x=21}^{25}{}_{25}C_x(0.8)^x(0.2)^{25-x}=0.579326$$

(c) X – daily demand of dessert

X ~ Po(7)

$$\Pr(X>5)=1-\sum_{x=0}^{5}\frac{e^{-7}7^x}{x!}=0.69929$$

Y – number of days out of 90, that the daily demand of dessert can't be fulfilled completely

Y~B(90, 0.69929)

Since n>30, np>5, nq>5 and 0.1<p<0.9, $Y\sim N\left(62.9363,4.35034^2\right)$ approximately

$$\Pr(Y\ge60)=\Pr(Y>59.5)\approx\Pr(Z>-0.79)=0.7852$$

4. (a) X – weekly revenue of a cafe (\$)

$$\bar{X}\sim N\left(75000,\frac{8000^2}{4}\right)$$

$\Pr\left(70000 < \bar{X} < 95000\right) = \Pr\left(-1.25 < Z < 5\right) \approx 0.8944$

(b) p – proportion of orders that are delivered late

$H_0:$ p = 0.3

$H_1:$ p < 0.3

$\alpha = 0.025$

Critical region: z < -1.96

$\hat{p} = \frac{14}{60}$

Under H_0, test statistic $z = \frac{14/60 - 0.3}{\sqrt{0.3 \times 0.7/60}} = -1.1269$

Decision: Do not reject H_0

(c) X – amount purchase ($) by a customer

$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ approximately

$H_0:\ \mu = 25$

$H_1:\ \mu < 25$

$\alpha = 0.05$

Critical region: z < -1.645

Under H_0, test statistic $z = \frac{24.57 - 25}{6.6/\sqrt{100}} = -0.6515$

Decision: Do not reject H_0

(d)

$H_0:$ historical pattern of sales prevails

$H_1:\ H_0$ false

$\alpha = 0.01$

Expected frequencies are 40, 40 and 20

Critical region: $\chi^2 > 9.210, \nu = 2$

Under H_0, test statistic $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 11.25$

Decision: Reject H_0

5. (a) p_i - proportion of people watching TV special in community *i*

$\hat{p}_1 - \hat{p}_2 \sim N\left(p_1 - p_2, \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}\right)$ approximately

$H_0 : p_1 = p_2$

$H_1 : p_1 \neq p_2$

$\alpha = 0.01$

Critical region: z < -2.576 and z > 2.576

$\hat{p}_1 = \frac{50}{250}$ $\hat{p}_2 = \frac{100}{300}$ $\hat{p} = \frac{150}{550}$

Under H_0, test statistic $z = \frac{\left(\frac{1}{5} - \frac{1}{3}\right) - 0}{\sqrt{\frac{3}{11} \times \frac{8}{11}\left(\frac{1}{250} + \frac{1}{300}\right)}} = -3.4960$

Decision: Reject H_0

(b) D – paired difference of sales amount

d: 16 23 9 8 4 5 -4 20 9 5

n=10, $\sum d = 95$, $\sum d^2 = 1493$

$\bar{d} = \frac{95}{10} = 9.5$, $s_d = \sqrt{\frac{10(1493) - 95^2}{10(10-1)}} = 8.1000$

$H_0 : \mu_d = 0$

$H_1 : \mu_d > 0$

$\alpha = 0.05$

Critical region: t > 1.833, $\nu = 9$

Under H_0, test statistic $t = \frac{9.5 - 0}{8.1/\sqrt{10}} = 3.7088$

Decision: Reject H_0

(c) H_0 : level of satisfaction and gender are independent

H_1 : level of satisfaction and gender are not independent

$\alpha = 0.05$

Critical region: $\chi^2 > 5.991, \nu = 2$

Expected frequencies:

	Unsatisfactory	Neutral	Satisfactory
Male	25.38	39.94	142.69
Female	35.62	56.06	200.31

Under H_0, test statistic $\chi^2 = \sum \frac{\left(O_{ij} - E_{ij}\right)^2}{E_{ij}} = 5.4330$

Decision: Do not reject H_0

6. (a) (i)

$$b = \frac{10(285625) - (1675)(1700)}{10(286400) - (1675)^2} = 0.1499$$

$$a = \frac{1700}{10} - 0.1499 \times \frac{1675}{10} = 144.8929$$

$$\hat{y} = 144.8929 + 0.1499x$$

(ii) $1 - R^2 = 1 - \frac{\left[10(285625) - (1675)(1700)\right]^2}{\left[10(286400) - (1675)^2\right]\left[10(289800) - (1700)^2\right]} = 0.8361$

(b) (i) $\hat{y} = 12.2006 - 0.14203x_1 + 0.28202x_2 + 0.619x_3 + 0.00000792x_4$

(ii)

$a = 236.557 - 138.327 = 98.23$

$b = 4$

$c = 81 - 4 - 1 = 76$

$d = 81 - 1 = 80$

$e = 138.327/4 = 34.5818$

$f = 98.23/76 = 1.2925$

$g = 34.5818/1.2925 = 26.7557$

(iii) $R^2 = \dfrac{138.327}{236.557} = 0.5848$

(iv) $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$

H_1 : at least one of $\beta_i \neq 0, i = 1, 2, 3, 4$

$\alpha = 0.05$

Critical region: f > 2.53, $\nu_1 = 4, \nu_2 = 76$

Under H_0, test statistic f = 26.7557

Decision: Reject H_0