AMA1501 Introduction to Statistics for Business
Mock Examination Paper 8 Outline Suggested Solution

1.

Class boundary	Class mark, x	Frequency, f	Score less than	Cum. Freq.
20.5 – 30.5	25.5	2	20.5	0
30.5 – 40.5	35.5	7	30.5	2
40.5 – 50.5	45.5	15	40.5	9
50.5 – 60.5	55.5	24	50.5	24
60.5 – 70.5	65.5	38	60.5	48
70.5 – 80.5	75.5	31	70.5	86
80.5 – 90.5	85.5	17	80.5	117
90.5 – 100.5	95.5	6	90.5	134
			100.5	140

(a)

$\sum f = 140 \quad \sum fx = 9170 \quad \sum fx^2 = 633835$

Mean = $\frac{9170}{140} = 65.5$

Standard deviation = $\sqrt{\frac{140(633835) - 9170^2}{140(140-1)}} = 15.4547$

Mode = $60.5 + \frac{38-24}{(38-24)+(38-31)}(70.5-60.5) = 67.1667$

(b)

$D_2 = 50.5 + \frac{28-24}{24}(60.5-50.5) = 52.1667 \ \ kg/m^2$

(c)

$\hat{p} = \left(\frac{60.5-55}{60.5-50.5} \times 24 + 38 + 31 + \frac{88-80.5}{90.5-80.5} \times 17 \right) \Big/ 140 = 0.6782$

A 95% C.I. for p is $0.6782 \pm 1.96\sqrt{\frac{0.6782(1-0.6782)}{140}}$, i.e. $0.6008 < p < 0.7556$

(d) $CV_{MT} = \dfrac{15.4547}{65.5} \times 100\% = 23.5950\%$

$CV_{AO} = \dfrac{17}{78} \times 100\% = 21.7949\%$

2.

(a) Pr(2 particular guests will not be seated on the same table)

$= \dfrac{{}_{20}C_{10} \times {}_{10}C_{10} - {}_{2}C_{2} \times {}_{18}C_{8} \times {}_{10}C_{10}}{{}_{20}C_{10}} = 0.7632$

(b) A: learn playing piano

B: learn playing violin

$\Pr(A) = 0.75 \quad \Pr(B) = 0.56 \quad \Pr(\bar{A} \cap \bar{B}) = 0.05$

(i) $\Pr\left[(A \cap \bar{B}) \cup (\bar{A} \cap B)\right] = \Pr(A \cup B) - \Pr(A \cap B)$

$= (1 - 0.05) - \left[0.75 + 0.56 - (1 - 0.05)\right] = 0.59$

(ii) $\Pr(\bar{A} \mid B) = 1 - \dfrac{0.36}{0.56} = \dfrac{5}{14}$

(iii) $\Pr(B \mid \bar{A}) = \dfrac{0.56 - 0.36}{1 - 0.75} = \dfrac{4}{5}$

(c) A – cloth is supplied by Supplier A

B – cloth is supplied by Supplier B

C – cloth is supplied by Supplier C

D – there are 2 nonconformities on a cloth of 2 m-sq.

$\Pr(A) = 0.4 \quad \Pr(B) = 0.35 \quad \Pr(C) = 0.25$

$\Pr(D \mid A) = \dfrac{e^{-3}(3)^2}{2!} = 0.2240$

$\Pr(D \mid B) = \dfrac{e^{-2}(2)^2}{2!} = 0.2707$

$\Pr(D \mid C) = \dfrac{e^{-4}(4)^2}{2!} = 0.1465$

$\Pr(A \mid D) = \dfrac{0.4 \times 0.2240}{0.4 \times 0.2240 + 0.35 \times 0.2707 + 0.25 \times 0.1465} = 0.4055$

3. (a) X – amount of daily sales ($), $X \sim N\left(4000, 1000^2\right)$

(i) $\Pr\left(2580 < X < 5500\right) = \Pr\left(-1.42 < Z < 1.5\right) = 0.8554$

(ii) $\Pr\left(X < a\right) = \Pr\left(Z < \dfrac{a-4000}{1000}\right) = 0.1 \Rightarrow \dfrac{a-4000}{1000} = -1.28 \Rightarrow a \approx \$2720$

(iii) $\Pr\left(\bar{X} > 3600\right) \approx \Pr\left(Z > -0.89\right) = 0.8133$

(b) X – number of people will be affected by the disease out of 100

$X \sim B\left(100, 0.004\right)$

As n>100 and p<0.1, Poission approximation to Binomial can be used

$\lambda = 1000 \times 0.004 = 4$

$$\Pr\left(4 \le X \le 6\right) = \sum_{x=4}^{6} \frac{e^{-4}4^x}{x!} = 0.4559$$

(c) X – number of customers wait for more than 5 minutes, out of 50

X ~ B(50, 0.22)

Since n>30, np>5, nq>5 and 0.1<p<0.9, $X \sim N\left(11, 8.58\right)$ approximately

$$\Pr\left(X < 20\right) = \Pr\left(X < 19.5\right) \approx \Pr\left(Z < 2.90\right) = 0.99813$$

4. (a)

$$1.96\sqrt{\frac{0.7 \times 0.3}{n}} = 0.04$$

$$n = 504.21$$

$$\therefore n = 505$$

(b)

$$s_p^2 = \frac{(16-1)8^2 + (14-1)10^2}{16+14-2} = 80.7143$$

A 99% confidence interval for $\mu_1 - \mu_2$ is

$$(76-82) \pm 2.763\sqrt{80.7143\left(\frac{1}{16} + \frac{1}{14}\right)}$$

i.e. $-15.0843 < \mu_1 - \mu_2 < 3.0843$

(c) p_1 - proportion of technical staff favour the changes

p_2 - proportion of administrative staff favour the changes

$\hat{p}_1 - \hat{p}_2 \sim N\left(p_1 - p_2, \frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}\right)$ approximately

$H_0 : p_1 = p_2$

$H_1 : p_1 > p_2$

$\alpha = 0.01$

Critical region: z > 2.33

$\hat{p}_1 = \frac{84}{120} \qquad \hat{p}_2 = \frac{62}{100} \qquad \hat{p} = \frac{146}{220}$

Under H_0, test statistic $z = \frac{\left(\frac{84}{120} - \frac{62}{100}\right) - 0}{\sqrt{\frac{146}{220} \times \frac{74}{220}\left(\frac{1}{120} + \frac{1}{100}\right)}} = 1.25$

Decision: Do not reject H_0

5. (a)

H_0 : purchasing behaviour of customers is the same as previous years

H_1 : H_0 false

$\alpha = 0.05$

Observed frequencies are 90, 275 and 135, with corresponding expected frequencies 125, 250 and 125 respectively.

Critical region: $\chi^2 > 5.991, \nu = 2$

Under H_0, test statistic $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 13.1$

Decision: Reject H_0

(b) D – paired difference of sales amount

d: 2 6 6 4 8 4 -1

n=7, $\sum d = 29$, $\sum d^2 = 173$

$\bar{d} = \frac{29}{7} = 4.1429$, $s_d = \sqrt{\frac{7(173) - 29^2}{7(7-1)}} = 2.9681$

$H_0: \ \mu_d = 0$

$H_1: \ \mu_d > 0$

$\alpha = 0.05$

Critical region: t > 1.943, $\nu = 6$

Under H_0, test statistic $t = \frac{4.1429 - 0}{2.9681/\sqrt{7}} = 3.6929$

Decision: Reject H_0

(c) H_0: grade of employee and opinion on program are independent

H_1: grade of employee and opinion on program are not independent

$\alpha = 0.05$

Critical region: $\chi^2 > 9.488, \nu = 4$

Expected frequencies:

	Disagree	Neutral	Agree
Senior	6.15	13.50	10.35
Middle	14.35	31.50	24.15
Worker	20.50	45.00	34.50

Under H_0, test statistic $\chi^2 = \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}} = 26.85$

Decision: Reject H_0

6. (a) (i)

$b = \frac{12(9716.62) - (226.7)(492.9)}{12(4552.45) - (226.7)^2} = 1.5013$

$a = \frac{492.9}{12} - 1.5013 \times \frac{226.7}{12} = 12.7127$

$\hat{y} = 12.7127 + 1.5013x$

(ii) When $x = 20$, $\hat{y} = 12.7127 + 1.5013(20) = 42.7390$ ($'000)

(b) (i) $\hat{y} = -124.382 + 0.296x_1 + 0.048x_2 + 1.306x_3 + 0.520x_4$

(ii)

$a = 9054 - 335.978 = 8718.022$

$b = 4$

$c = 25 - 4 - 1 = 20$

$d = 25 - 1 = 24$

$e = 8718.022/4 = 2179.5505$

$f = 335.978/20 = 16.7989$

$g = 2179.5505/16.7989 = 129.7410$

(iii) $R^2 = 1 - \dfrac{335.978}{9054} = 0.9629$

(iv) $H_0 : \beta_3 = 0$

$H_1 : \beta_3 \neq 0$

$\alpha = 0.05$

Critical region: t < -2.086 and t > 2.086, $\nu = 20$

Under H_0, test statistic

$t = \dfrac{1.306}{0.164} = 7.96$

Decision: Reject H_0