

Mean of Poisson distribution

$$E[X] = \sum_{x=0}^{\infty} x \cdot \Pr(X = x)$$

$$= \sum_{x=0}^{\infty} x \cdot \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= \sum_{x=1}^{\infty} x \cdot \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= \lambda \cdot \sum_{x=1}^{\infty} \frac{e^{-\lambda} \lambda^{x-1}}{(x-1)!}$$

Let $y = x - 1$

$$= \lambda \cdot \sum_{y=0}^{\infty} \frac{e^{-\lambda} \lambda^y}{y!}$$

$$= \underline{\underline{\lambda}}$$

Variance of Poisson distribution

$$\begin{aligned}\text{Var}(X) &= E[X^2] - (E[X])^2 \\ &= E[(X)(X-1)] + E[X] - (E[X])^2\end{aligned}$$

$$\begin{aligned}E[(X)(X-1)] &= \sum_{x=0}^{\infty} x(x-1) \cdot \frac{e^{-\lambda} \lambda^x}{x!} \\ &= \sum_{x=2}^{\infty} \frac{e^{-\lambda} \lambda^x}{(x-2)!} \\ &= \lambda^2 \sum_{x=2}^{\infty} \frac{e^{-\lambda} \lambda^{x-2}}{(x-2)!}\end{aligned}$$

Let $y = x-2$

$$\begin{aligned}&= \lambda^2 \sum_{y=0}^{\infty} \frac{e^{-\lambda} \lambda^y}{y!} \\ &= \lambda^2\end{aligned}$$

$$\therefore \text{Var}(X) = \lambda^2 + \lambda - \lambda^2$$

$$= \underline{\underline{\lambda}}$$