Mean of Poisson distribution

$$E[X] = \sum_{x=0}^{\infty} x \cdot \Pr(X = x)$$

$$= \sum_{x=0}^{\infty} x \cdot \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= \sum_{x=1}^{\infty} x \cdot \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= \lambda \cdot \sum_{x=1}^{\infty} \frac{e^{-\lambda}\lambda^{x-1}}{(x-1)!}$$

Let $y = x - 1$

$$= \lambda \cdot \sum_{y=0}^{\infty} \frac{e^{-\lambda}\lambda^{y}}{y!}$$

$$= \underline{\underline{\lambda}}$$

Variance of Poisson distribution

$$Var(X) = E[X^2] - (E[X])^2$$

$$= E[(X)(X-1)] + E[X] - (E[X])^2$$

$$E[(X)(X-1)] = \sum_{x=0}^{\infty} x(x-1) \cdot \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=2}^{\infty} \frac{e^{-\lambda}\lambda^x}{(x-2)!}$$

$$= \lambda^2 \sum_{x=2}^{\infty} \frac{e^{-\lambda}\lambda^{x-2}}{(x-2)!}$$

Let $y = x$-2

$$= \lambda^2 \sum_{y=0}^{\infty} \frac{e^{-\lambda}\lambda^y}{y!}$$

$$= \lambda^2$$

$$\therefore Var(X) = \lambda^2 + \lambda - \lambda^2$$

$$= \underline{\underline{\lambda}}$$