

Sampling Distribution of Difference of Means

$$\begin{aligned}\mu_{\bar{X}_1 - \bar{X}_2} &= E[\bar{X}_1 - \bar{X}_2] \\ &= E[\bar{X}_1] - E[\bar{X}_2] \\ &= \mu_{\bar{X}_1} - \mu_{\bar{X}_2} \\ &= \mu_1 - \mu_2\end{aligned}$$

$$\begin{aligned}\sigma_{\bar{X}_1 - \bar{X}_2}^2 &= \text{Var}(\bar{X}_1 - \bar{X}_2) \\ &= \text{Var}(\bar{X}_1) + \text{Var}(\bar{X}_2) && \because \bar{X}_1 \text{ and } \bar{X}_2 \text{ are independent} \\ &= \sigma_{\bar{X}_1}^2 + \sigma_{\bar{X}_2}^2 \\ &= \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\end{aligned}$$