Sampling Distribution of Means

$$\mu_{\overline{X}} = E\left[\overline{X}\right]$$

$$= E\left[\frac{\sum X}{n}\right]$$

$$= \frac{1}{n}E\left[\sum X\right]$$

$$= \frac{1}{n}E\left[X_1 + X_2 + \ldots.. + X_n\right]$$

$$= \frac{1}{n}\left(E\left[X_1\right] + E\left[X_2\right] + \ldots.. + E\left[X_n\right]\right)$$

$$= \frac{1}{n}(n\mu) \qquad \because\ E\left[X_i\right] = \mu$$

$$= \mu$$

$$\sigma^2_{\overline{X}} = Var\left(\overline{X}\right)$$

$$= Var\left(\frac{\sum X}{n}\right)$$

$$= \frac{1}{n^2}Var\left(\sum X\right)$$

$$= \frac{1}{n^2}Var\left(X_1 + X_2 + \ldots.. + X_n\right)$$

$$= \frac{1}{n^2}\left[Var\left(X_1\right) + Var\left(X_2\right) + \ldots.. + Var\left(X_n\right)\right]$$

$\because$ $X_1, X_2, \ldots.., X_n$ are independent

$$= \frac{1}{n^2}\left(n\sigma^2\right) \qquad \because\ Var\left(X_i\right) = \sigma^2$$

$$= \frac{\sigma^2}{n}$$