Department of Applied Mathematics
AMA1501 Introduction to Statistics for Business
Homework 2015/2016 Semester 2 Suggested outline solution

1(a) Class marks: 125, 375, 625, 875, 1125, 1375, 1750, 2250

$\sum fx = 153875 \quad \sum fx^2 = 200546875$ n=140

Mean = $\bar{x} = \frac{153875}{140} = \$1099.11$

Standard deviation = $s = \sqrt{\frac{200546875 - 153875^2/140}{139}} = \sqrt{226055.8517} = \$475.45$

Median=$1000 + \frac{140(0.5) - 55}{36}(250) = \$1104.17$

1(b) Coefficient of variation of invoice amount of customers using co-branded credit card is $\frac{475.45}{1099.11} \times 100\% = 43.26\%$.

Coefficient of variation of invoice amount of customers using other credit cards is $\frac{450}{900} \times 100\% = 50\%$.

1(c) $\$0 + \$50 \times \frac{17+23+36+28}{140} + \$100 \times \frac{15+6}{140} = \$52.14$

1(d) $\hat{p} = \left(\frac{1000-800}{1000-750} \times 23 + 36 + 28 + \frac{1600-1500}{2000-1500} \times 15 \right) \Big/ 140 = \frac{85.4}{140} = 0.61$

Let X be the number of invoices have invoice amount between \$800 and \$1600 out of 6, $X \sim B(6, 0.61)$

$\Pr(X \geq 3) = 1 - \sum_{x=0}^{2} \binom{6}{x} (0.61)^x (1-0.61)^{6-x} = 0.8343$

2(a) $\frac{C_0^9 \times C_8^8 + C_1^9 \times C_7^8 + C_2^9 \times C_6^8}{C_8^{17}} = 0.04447$

2(b) Let A be the event that strawberry will be contained in the appetizer

D be the event that strawberry will be contained in the dessert

P(A) = 0.4, P(D) = 0.32, $P(A \cup D) = 0.6$

2(b)(i) $P(A \cup D) = P(A) + P(D) - P(A \cap D)$

$0.6 = 0.4 + 0.32 - P(A \cap D)$

$P(A \cap D) = 0.12$

2(b)(ii) $P(\bar{A} \mid D) = 1 - P(A \mid D) = 1 - \frac{P(A \cap D)}{P(D)} = 1 - \frac{0.12}{0.32} = 0.625$

2(b)(iii) $\Pr(D \mid \bar{A}) = \frac{\Pr(D \cap \bar{A})}{\Pr(\bar{A})} = \frac{\Pr(D) - \Pr(D \cap A)}{1 - \Pr(A)} = \frac{0.32 - 0.12}{1 - 0.4} = \frac{1}{3}$

2(b)(iv) Let S be the event that the randomly selected day is Saturday

$$\Pr\left(A\cap D\middle|S\right)=0.42$$

$$\Pr\left(S\middle|A\cap D\right)=\frac{\Pr\left(A\cap D\middle|S\right)\Pr\left(S\right)}{\Pr\left(A\cap D\right)}=\frac{0.42\times1/7}{0.12}=\frac{1}{2}$$

2(c) Let A be the event that the investment of the company in Country A
B be the event that the investment of the company in Country B
C be the event that the investment of the company in Country C
R2 be the event that the monthly return is greater than 2%

$$P(A)=0.3,\ P(B)=0.3,\ P(C)=0.4$$

$$P(R2\,|\,A)=0.13,\ P(R2\,|\,B)=0.09,\ P(R2\,|\,C)=0.08$$

By Baye's theorem, we have

$$P(C\,|\,R2)=\frac{P(R2\,|\,C)P(C)}{P(R2\,|\,A)P(A)+P(R2\,|\,B)P(B)+P(R2\,|\,C)P(C)}$$

$$=\frac{0.08\times0.4}{0.13\times0.3+0.09\times0.3+0.08\times0.4}=0.32653$$

3(a)(i) Let X be the daily sales amount of the shop.

$$X\sim N(40000,8000^2)$$

$$P(24000<X<52000)=P\left(\frac{24000-40000}{8000}<Z<\frac{52000-40000}{8000}\right)$$

$$=P(-2<Z<1.5)=1-0.0228-0.0668=0.9104$$

3(a)(ii) Let m be the daily sales amount exceeded by 5% of daily sales amounts.

$$P(X>m)=P\left(Z>\frac{m-40000}{8000}\right)=0.05$$

$$\frac{m-40000}{8000}=1.645 \qquad m=\$53160$$

3(a)(iii)

$$P(X>34400)=P\left(Z>\frac{34400-40000}{8000}\right)=P(Z>-0.7)=0.758$$

Let Y be the number of days whose daily sales amounts are more than \$34,400 each out of 100 days.

$$Y\sim b(100,0.758)$$

$$n=100>30 \qquad 0.1<p<0.9$$

$$np=100(0.758)=75.8>5 \qquad nq=100(0.242)=24.2>5$$

Normal approximation can be used.

$$P(Y\geq70)\approx P(Y>69.5)=P\left(Z>\frac{69.5-75.8}{\sqrt{100(0.758)(0.242)}}\right)=P(Z>-1.47)=0.9292$$

3(b)(i) Let X be the demand of the super-deluxe suites of the hotel per day.

$X \sim \mathbf{Po}(3)$

$$P(X \le 4) = e^{-3}(\frac{3^0}{0!}+\frac{3^1}{1!}+\frac{3^2}{2!}+\frac{3^3}{3!}+\frac{3^4}{4!}) = 0.8153$$

3(b)(ii)

$$P(X \le 4 \mid X \ge 2) = \frac{P(2 \le X \le 4)}{P(X \ge 2)} = \frac{e^{-3}(\frac{3^2}{2!}+\frac{3^3}{3!}+\frac{3^4}{4!})}{1-e^{-3}(\frac{3^0}{0!}+\frac{3^1}{1!})} = 0.7693$$

4(a) Let $\overline{X}$ be the average monthly tuition fee of the 15 kindergarteners.

$$\overline{X} \sim N(3200, \frac{1000^2}{15})$$

$$\Pr(2500 < \overline{X} < 3000) = \Pr(\frac{2500-3200}{1000/\sqrt{15}} < Z < \frac{3000-3200}{1000/\sqrt{15}}) \approx \Pr(-2.71 < Z < -0.77)$$

$$= 0.2206 - 0.00336 = 0.21724$$

4(b)(i) $\sum x = 32670$, $\sum x^2 = 107149900$, n=10

$$\bar{x} = 32670/10 = 3267 \ , s = \sqrt{\frac{107149900 - 32670^2/10}{9}} = \sqrt{\frac{417010}{9}} = 215.2543715$$

A 95% confidence interval for the mean monthly tuition fee is

$3267 \pm 2.262(215.2543/\sqrt{10})$ i.e. $\$3113.0270 < \mu < \$3420.9730$

4(b)(ii)

$$1.96 \times \frac{215.2431715}{\sqrt{n}} \le 50$$

$$n \ge \left(\frac{1.96 \times 215.2437}{50}\right) = 71.19$$

4(c) Let p be the proportion of five-year-old children who are learning any musical instruments.
A 99% confidence interval for p is

$\frac{105}{180} \pm 2.576\sqrt{\frac{105}{180} \times \frac{75}{180}/180}$, i.e. $0.4887 < p < 0.6780$