Department of Applied Mathematics
AMA1501 Introduction to Statistics for Business
Homework 2016/2017 Semester 1 Suggested outline solution

1(a)

Total expenditure ($)	Class boundary ($)	Classmarks	Number of VIP customers	cf (< l2)
0 – 199	-0.5 – 199.5	99.5	5	5
200 – 299	199.5 – 299.5	249.5	7	12
300 – 399	299.5 – 399.5	349.5	13	25
400 – 499	399.5 – 499.5	449.5	28	53
500 – 599	499.5 – 599.5	549.5	16	69
600 – 799	599.5 – 799.5	699.5	8	77
800 – 999	799.5 – 999.5	899.5	3	80

$\sum f = 80 \quad \sum fx = 36460 \quad \sum fx^2 = 18903520$

Mean = $\bar{x} = \dfrac{36460}{80} = \$455.75$

Mode $= 399.5 + \dfrac{28-13}{(28-13)+(28-16)}(499.5-399.5) = \$455.05556$

Standard deviation = $s = \sqrt{\dfrac{80(18903520)-36460^2}{80(80-1)}} = \$170.1404855$

1(b) $D_8 = 499.5 + \dfrac{80\times 0.8-53}{16}(599.5-499.5) = \$568.25$

1(c) proportion exceed \$380 $= \left(\dfrac{399.5-380}{399.5-299.5}\times 13+28+16+8+3\right)\Big/80 = 0.71919$

1(d) P(spend at most \$250)= $\left(5+\dfrac{250-199.5}{299.5-199.5}\times 7\right)\Big/80 = 0.1067$

Let X be the no. of VIP customers with total amounts of expenditure at most \$250 out of the 20 VIP customers,

$X \sim b(20, 0.1067)$

P(less than 2 VIP customers have total amounts at most \$250)

$= \sum_{x=0}^{1}\binom{20}{x}(0.1067)^x(1-0.1067)^{20-x} = 0.3549$

2(a)(i) Number of selections = ${}_7C_2 \times {}_6C_1 + {}_7C_3 \times {}_6C_0 = 161$

2(a)(ii) Required probability = $\dfrac{{}_5C_2 \times {}_4C_1 + {}_5C_3 \times {}_4C_0}{161} = \dfrac{50}{161}$

2(b) Let GA be the event that the customer would buy game A

GB be the event that the customer would buy game B

$P(GA \cup GB) = 0.78, \quad P(GA) = 0.55, \quad P(GB) = 0.47$

2(b)(i) $P(GA \cup GB) = P(GA) + P(GB) - P(GA \cap GB)$

$P(GA \cap GB) = 0.55 + 0.47 - 0.78 = 0.24$

2(b)(ii) $P(GA\,|\,\overline{GB}) = \dfrac{P(GA \cap \overline{GB})}{P(\overline{GB})} = \dfrac{P(GA) - P(GA \cap GB)}{1 - P(GB)} = \dfrac{0.55 - 0.24}{1 - 0.47} = 0.5849$

2(b)(iii) Let GC be the event that the customer would buy game C

$P(GC) = 0.2,\ \ P(\overline{GC}\,|\,GA) = 0.83$

$P(GC\,|\,GA) = 1 - P(\overline{GC}\,|\,GA) = 0.17$

$P(GA \cap GC) = P(GC\,|\,GA)P(GA) = 0.17 \times 0.55 = 0.0935$

$$\begin{aligned} P(\overline{GA \cup GC}) &= 1 - P(GA \cup GC) \\ &= 1 - P(GA) - P(GC) + P(GA \cap GC) \\ &= 1 - 0.55 - 0.2 + 0.0935 = 0.3435 \end{aligned}$$

2(c) Let A1 be the event that the apple comes from source S1
A2 be the event that the apple comes from source S2
A3 be the event that the apple comes from source S3
A4 be the event that the apple comes from source S4
R be the event that the apple is rotten.

$P(A1) = 0.3,\ \ P(A2) = 0.2,\ \ P(A3) = 0.1,\ \ P(A4) = 0.4$

$P(R\,|\,A1) = 0.05,\ \ P(R\,|\,A2) = 0.06,\ \ P(R\,|\,A3) = 0.03,\ \ P(R\,|\,A4) = 0.04$

Since A1, A2, A3 and A4 are mutually exclusive and collectively exhaustive, it follows from Bayes' theorem that

$$P(A4\,|\,R) = \frac{P(R\,|\,A4)P(A4)}{P(R\,|\,A1)P(A1) + P(R\,|\,A2)P(A2) + P(R\,|\,A3)P(A3) + P(R\,|\,A4)P(A4)}$$

$$= \frac{0.04(0.4)}{0.05(0.3) + 0.06(0.2) + 0.03(0.1) + 0.04(0.4)} = \frac{0.016}{0.046} = 0.3478$$

3(a)(i) Let X be the duration of a long distance call.

$X \sim N(240, 40^2)$

$P(160 < X < 330) = P(\frac{160 - 240}{40} < Z < \frac{330 - 240}{40}) = P(-2 < Z < 2.25) = 0.965$

3(a)(ii) Let m be the duration exceeded by 12.5% of durations of long distance calls.

$P(X > m) = 0.125 \qquad P(Z > \frac{m - 240}{40}) = 0.125$

$\frac{m - 240}{40} = 1.15 \qquad m = 286$ seconds

3(a)(iii) $P(X > 220) = P(Z > \frac{220 - 240}{40}) = P(Z > -0.5) = 0.6915$

Let Y be the number of calls with duration more than 220 seconds each out of 100.

$Y \sim b(100, 0.6915)$

$n = 100 > 30 \quad 0.1 < p < 0.9$

$np = 100(0.6915) = 69.15 > 5 \qquad nq = 100(0.3085) = 30.85 > 5$

Normal approximation can be used.

$P(Y > 70) \approx P(Y > 70.5) = P(Z > \frac{70.5 - 69.15}{\sqrt{100(0.6915)(0.3085)}}) = P(Z > 0.29) = 0.3859$

3(b)(i) Let X be the number of chocolate chips on a cookie.

$X \sim \mathbf{Po}(6)$

$$P(X<4)=e^{-6}\left(\frac{6^0}{0!}+\frac{6^1}{1!}+\frac{6^2}{2!}+\frac{6^3}{3!}\right)=0.151204\approx 0.1512$$

3(b)(ii)

$$P(X=3\mid X<4)=\frac{P(X=3)}{P(X<4)}=\frac{0.089235}{0.151204}=0.590164$$

Required Probability = $0.590164^5=0.0716$

4(a) Let X be the price of mobile phones PolyU students pay,

$$P(\overline{X}<2600)=P\left(\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}<\frac{2600-2500}{400/\sqrt{45}}\right)=P(Z<1.68)=1-0.0465=0.9535$$

4(b)(i) $\sum x=77367, \sum x^2=778608703$

$\bar{x}=77367/8=9670.875$, $s=\sqrt{\dfrac{778608703-77367^2/8}{7}}$ =2084.024845,

$\nu=n-1=7$, $t=2.365$

A 95% confidence interval for the mean price of all notebook/tablet computers with the i7 processor is

$9670.875\pm 2.365(2084.024845/\sqrt{8})$

i.e. $7928.309772<\mu<11413.44023$

4(b)(ii)

$$1.96\times\frac{2084.024845}{\sqrt{n}}\le 500\Rightarrow n\ge 66.74\approx 67$$

4(c) Let p be the proportion of PolyU staff who have traveled overseas in 2014

n=200, $p=146/200=0.73$

A 99% confidence interval for the true proportion of PolyU staff who have traveled overseas in 2014 is

$$0.73\pm 2.576\sqrt{\frac{0.73(0.27)}{200}}$$

i.e. $0.6491325<p<0.8108675$