**Department of Applied Mathematics**
**AMA1501 Introduction to Statistics for Business**
**Homework 2016/2017 Semester 2 Suggested outline solution**

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| Hotel room rates ($) | Frequency | Classmark | $fx$ | $fx^2$ | cf |
|---|---|---|---|---|---|
| 200-300 | 5 | 250 | 1250 | 312500 | 5 |
| 300-500 | 20 | 400 | 8000 | 3200000 | 25 |
| 500-700 | 18 | 600 | 10800 | 6480000 | 43 |
| 700-900 | 35 | 800 | 28000 | 22400000 | 78 |
| 900-1100 | 16 | 1000 | 16000 | 16000000 | 94 |
| 1100-1500 | 4 | 1300 | 5200 | 6760000 | 98 |
| 1500-2500 | 2 | 2000 | 4000 | 8000000 | 100 |
| | 100 | | 73250 | 63152500 | |

1(a) Mean $= \bar{x} = \frac{73250}{100} = \$732.50$

$$\text{Mode} = 700 + \frac{35-18}{(35-18)+(35-16)}(900-700) = \$794.44$$

$$\text{Standard Deviation} = s = \sqrt{\frac{63152500 - 73250^2/100}{100-1}} = \$309.7225$$

1(b) $D_3 = 500 + \frac{30-25}{18}(700-500) = \$555.5556$

1(c) No. of tourists who spend \$750 to \$1200

$$= 35(\frac{900-750}{200}) + 16 + 4(\frac{1200-1100}{400}) = 43.25$$

1(d) From the frequency distribution table,

P(hotel room rate less than \$900)$= \frac{5+20+18+35}{100} = 0.78$

Let X be the no. of tourists who are spending less than \$900 for a hotel room per night out of 10 tourists,

$X \sim b(n=10, p=0.78)$

$P(X=5) = {}_{10}C_5 0.78^5 (1-0.78)^5 = 0.037496174$

2(a) $$1 - \frac{\binom{11}{10}\binom{15}{0}}{\binom{26}{10}} - \frac{\binom{11}{9}\binom{15}{1}}{\binom{26}{10}} - \frac{\binom{11}{8}\binom{15}{2}}{\binom{26}{10}} = 1 - \frac{11+825+17325}{5311735} = 0.99658$$

2(b) Let A be the event that an individual traveler will go to Park A
B be the event that an individual traveler will go to Park B
C be the event that an individual traveler will go to Park C

$P(A) = 0.54,\ P(C) = 0.43,\ P(A \cap B) = 0.2,\ P(A \cup B) = 0.88$

2(b)(i) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$0.88=0.54+P(B)-0.2$

$P(B)=0.54$

2(b)(ii) $P(\overline{B}\mid\overline{A})=\dfrac{P(\overline{B}\cap\overline{A})}{P(\overline{A})}=\dfrac{P(\overline{A\cup B})}{1-P(A)}=\dfrac{1-P(A\cup B)}{1-0.54}=\dfrac{1-0.88}{0.46}=0.2609$

2(b)(iii) $P(\overline{A}\mid C)=0.37,\ P(A\mid C)=1-0.37=0.63$

$P(A\cap C)=P(A\mid C)P(C)=0.63\times 0.43=0.2709$

$P(A\cap\overline{C})=P(A)-P(A\cap C)=0.54-0.2709=0.2691$

2(c) Let D be the event that the applicant graduated from the KLM University

S be the event that the application is successful

$P(D)=0.28,\ P(S\mid D)=0.31,\ P(S\mid\overline{D})=0.18$

$$P(D\mid S)=\frac{P(S\mid D)P(D)}{P(S\mid D)P(D)+P(S\mid\overline{D})P(\overline{D})}$$

$$=\frac{0.31\times 0.28}{0.31\times 0.28+0.18\times(1-0.28)}=0.4011$$

3(a)(i) Let X be the test score

$X\sim N\left(75,8^2\right)$

$$\Pr\left(65<X<82\right)=\Pr\left(-1.25<Z<0.875\right)$$

$$=1-0.1056-\left(0.1922+0.1894\right)/2=0.7036$$

3(a)(ii) Let a be the required lowest score

$$\Pr\left(X>a\right)=0.05\Rightarrow\frac{a-75}{8}=1.645$$

$a=75+1.645\times 8=88.16$

Lowest score to get the certificate of merit is 89.

3(a)(iii) $$\Pr\left(65<X<82\middle|X>62\right)=\frac{\Pr\left(65<X<82\cap X>62\right)}{\Pr\left(X>62\right)}=\frac{\Pr\left(65<X<82\right)}{\Pr\left(X>62\right)}$$

$$=\frac{0.7036}{\Pr\left(Z>-1.625\right)}=\frac{0.7036}{1-\left(0.0526+0.0516\right)/2}=0.7423$$

3(a)(iv) $200\times\Pr\left(X<65\right)=200\times 0.1056=21.12$

3(b) X – number of customers order special cake, out of 70

X~B(70,0.7)

As n>30, 0.1<p<0.9, np>5 & nq>5, normal approximation to binomial is used.

$$\Pr\left(X\le 50\right)\approx\Pr\left(Z\le\frac{50.5-70\times 0.7}{\sqrt{70\times 0.7\times 0.3}}\right)\approx\Pr\left(Z\le 0.39\right)=1-0.3483=0.6517$$

3(c) X – number of failures of a computer system in a period of 2 weeks

X~Po(2)

$$\Pr(X > 3) = 1 - \sum_{x=0}^{3} \frac{e^{-2}2^{x}}{x!} = 0.1429$$

4(a) Let $X$ be the weight of the energy bar.

$$\bar{X} \sim N(20, \frac{2^2}{24})$$

$$P(\bar{X} < \frac{460}{24}) = P(Z < \frac{460/24 - 20}{2/\sqrt{24}}) = P(Z < -2.04) = 0.0207$$

4(b)(i) $\sum x = 26140 \qquad \sum x^2 = 85670000$

$$\bar{x} = 26140/10 = 2614 \quad s = \sqrt{\frac{85670000 - 26140^2/10}{9}} = 1388.0458$$

A 90% confidence interval for the mean annual dental expenses is

$(2614) \pm 1.833 \frac{1388.0458}{\sqrt{10}}$, that is, $\$1809.4255 < \mu < \$3418.5745$

4(b)(ii) $1.645 \times \frac{1388.0458}{\sqrt{n}} \le 300 \qquad n \ge \left(\frac{1.645 \times 1388.0458}{300}\right)^2 = 57.93$

4(c) A 95% confidence interval for the true proportion of the claims that are paid within 2 months

$\frac{80}{200} \pm 1.96\sqrt{\frac{(80/200)(120/200)}{200}}$, that is, $0.3321 < p < 0.4679$