Department of Applied Mathematics
AMA1501 Introduction to Statistics for Business
Homework 2017/2018 Semester 1 Suggested outline solution

1(a) $\bar{x}=\frac{\sum fx}{\sum f}=\frac{19\times1000+30\times3000+36\times5000+18\times7000+11\times9000+6\times11000}{120}$

$=\frac{580000}{120}=\$4833.3333$

$\text{Mode}=L_1+\frac{d_1}{d_1+d_2}(L_2-L_1)=4000+(\frac{6}{6+18})(6000-4000)=\$4500$

$s=\sqrt{\frac{\sum fx^2-(\sum fx)^2/n}{n-1}}$

$=\sqrt{\frac{19\times1000^2+30\times3000^2+36\times5000^2+18\times7000^2+11\times9000^2+6\times11000^2-\frac{580000^2}{120}}{119}}$

$=\sqrt{\frac{3688000000-580000^2/120}{119}}=\$2726.5681$

1(b)

Monthly Expenses ($)	No. of retirees	Cumulative Frequency
0 but less than 2000	19	19
2000 but less than 4000	30	49
4000 but less than 6000	36	85
6000 but less than 8000	18	103
8000 but less than 10000	11	114
10000 but less than 12000	6	120

$P_{45}=4000+\frac{120*0.45-49}{36}(6000-4000)=\$4277.7778$

1(c)

Proportion = $\frac{36*\frac{500}{2000}+18+11+6}{120}=\frac{44}{120}=0.3667$

1(d) Let X be the no. of retirees with monthly expenses over \$5500

$X\sim b(n=10,p=\frac{44}{120})$, x=0,1,2,…,10

$P(X=4)=\binom{10}{4}\left(\frac{44}{120}\right)^4\left(\frac{76}{120}\right)^6=0.244962211$

2(a)(i) $\frac{C_4^7\times C_2^8}{C_6^{20}}=0.02528$

2(a)(ii) $\frac{P_4^4\times P_3^3}{P_6^6}=0.2$

2(b) Let M be the event that a citizen is a male

T be the event that the citizen had involved in a minor traffic accident

$P(M)=0.54, \quad P(T\,|\,M)=0.72, \quad P(T)=0.8$

2(b)(i) $P(T\cap M)=P(M)P(T\,|\,M)=0.54\times 0.72=0.3888$

$P(T\cup M)=P(T)+P(M)-P(T\cap M)=0.8+0.54-0.3888=0.9512$

2(b)(ii) $P(T\,|\,\overline{M})=\dfrac{P(T\cap\overline{M})}{P(\overline{M})}=\dfrac{P(T)-P(T\cap M)}{1-0.54}=\dfrac{0.8-0.3888}{0.46}=0.8939$

2(b)(iii) Let Y be the event that a citizen is under 30 years old.

$P(Y\,|\,M)=0.28, \quad P(Y\,|\,\overline{M})=0.34$

$P(Y)=P(M)P(Y\,|\,M)+P(\overline{M})P(Y\,|\,\overline{M})$

$=0.54\times 0.28+(1-0.54)\times 0.34$

$=0.3076$

2(c) Let PF be the event that the cargo is shipped through pier F
PG be the event that the cargo is shipped through pier G
PH be the event that the cargo is shipped through pier H
CJ be the event that the cargo is shipped to Hong Kong

$P(PF)=0.28, \quad P(PG)=0.37, \quad P(PH)=0.35$

$P(CJ\,|\,PF)=0.3, \quad P(CJ\,|\,PG)=0.2, \quad P(CJ\,|\,PH)=0.15$

$$P(PH\,|\,CJ)=\frac{P(CJ\,|\,PH)P(PH)}{P(CJ\,|\,PF)P(PF)+P(CJ\,|\,PG)P(PG)+P(CJ\,|\,PH)P(PH)}$$

$$=\frac{0.15\times 0.35}{0.3\times 0.28+0.2\times 0.37+0.15\times 0.35}$$

$=0.2494$

3(a)(i) Let X be the employee yearly turnover rate of a company.

$X\sim N(8,1.6^2)$

$$P(4.8<X<10.4)=P\left(\frac{4.8-8}{1.6}<Z<\frac{10.4-8}{1.6}\right)=P(-2<Z<1.5)$$

$=1-0.0228-0.0668=0.9104$

3(a)(ii) Let m be the turnover rate that just exceeds that of 20% of the companies.

$P(X<m)=0.2 \qquad P\left(Z<\dfrac{m-8}{1.6}\right)=0.2$

$\dfrac{m-8}{1.6}=-0.84 \qquad m=6.656\%$

3(a)(iii) $P(X>6\,|\,X<10)=\dfrac{P(6<X<10)}{P(X<10)}$

$$P(6<X<10)=P\left(\frac{6-8}{1.6}<Z<\frac{10-8}{1.6}\right)=P(-1.25<Z<1.25)$$

$=1-2(0.1056)=0.7888$

$P(X<10)=P(Z<1.25)=1-0.1056=0.8944$

$P(X>6\,|\,X<10)=\dfrac{0.7888}{0.8944}=0.8819$

3(b)(i) Let X be the number of service enquires received per hour.

$X \sim \mathbf{Po}(5)$

$$P(X \geq 5) = 1 - e^{-5}\left(\frac{5^0}{0!} + \frac{5^1}{1!} + \frac{5^2}{2!} + \frac{5^3}{3!} + \frac{5^4}{4!}\right) = 1 - 0.4405 = 0.5595$$

3(b)(ii) Let Y be the number of hours having more than 4 enquires each among 80 hours.

$Y \sim b(80, 0.5595)$

$n = 80 > 30$, $0.1 < p < 0.9$

$np = 80(0.5595) = 44.76 > 5$, $nq = 80(0.4405) = 35.24 > 5$

Normal approximation can be used.

$$P(Y \geq 50) \approx P(Y > 49.5) = P\left(Z > \frac{49.5 - 44.76}{\sqrt{80(0.5595)(0.4405)}}\right)$$

$$\approx P(Z > 1.07) = 0.1423$$

4(a) Let X be the time spent by voters in a polling station at district D (minutes).

$X \sim N\left(5, 1.2^2\right)$

$\overline{X} \sim N\left(5, \frac{1.2^2}{10}\right)$

$$\Pr\left(\frac{42}{10} < \overline{X} < \frac{52}{10}\right) = \Pr\left(\frac{4.2 - 5}{1.2/\sqrt{10}} < Z < \frac{5.2 - 5}{1.2/\sqrt{10}}\right) \approx \Pr\left(-2.11 < Z < 0.53\right)$$

$$= 1 - 0.0174 - 0.2981 = 0.6845$$

4(b)(i) $\sum x = 6517, \sum x^2 = 3823043$

$\overline{x} = 6517/12 = 543.0833$, $s = \sqrt{\frac{3823043 - 6517^2/12}{12 - 1}} = 160.615$

$\nu = n - 1 = 11$, $t = 2.201$

A 95% confidence interval for the mean expenditure at the online shop in the last month among all customers is

$$543.0833 \pm 2.201 \times \frac{160.615}{\sqrt{12}}$$

i.e. $441.0334 < \mu < 645.1332$

4(b)(ii) $2.576 \times \frac{160.615}{\sqrt{n}} \leq 20 \Rightarrow n \geq 427.904 \approx 428$

4(c) Let p be the proportion of voters who will vote for candidate A

n=500, $\hat{p} = 380/500 = 0.76$

A 95% confidence interval for the true proportion of voters who will vote for candidate A is

$0.76 \pm 1.96\sqrt{\frac{0.76(0.24)}{500}}$, i.e. $0.7226 < p < 0.7974$