Department of Applied Mathematics
AMA1501 Introduction to Statistics for Business/ AMA1502 Introduction to Statistics
Homework 2017/2018 Semester 2 Suggested outline solution

1

Average monthly spending less than ($)	Cumulative frequency
0	0
200	6
400	17
600	35
800	68
1000	120
1200	160

1(a) $\bar{x} = \frac{\sum fx}{\sum f} = \frac{100\times6+300\times11+500\times18+700\times33+900\times52+1100\times40}{160}$

$= \frac{126800}{160} = \$792.5$

Median $= 800 + \frac{160\times0.5-68}{52}(1000-800) = \$846.1538462$

$s = \sqrt{\frac{100^2\times6+300^2\times11+500^2\times18+700^2\times33+900^2\times52+1100^2\times40-(126800^2/160)}{160-1}}$

$= \sqrt{\frac{112240000-126800^2/160}{159}} = \$271.8559552$

1(b) $SK_2 = \frac{3(Mean-Median)}{Std.Dev.} = \frac{3\times(792.5-846.15385462)}{271.8559552} = -0.592$

The distribution is skewed to the left.

1(c) $D_8 = 1000 + \frac{160\times0.8-120}{40}(1200-1000) = \$1040$

1(d) Estimated no. of people = $52\times\frac{1000-850}{1000-800}+40\times\frac{1100-1000}{1200-1000} = 59$

2(a) $\frac{{}_1P_1\times{}_{19}P_1\times{}_1C_1\times{}_{49}C_3}{{}_{20}P_2\times{}_{50}C_4} = \frac{350056}{87514000} = \frac{1}{250}$

2(b) Let A be the event of selected employee is aged 45 or below.

Let B be the event of selected employee has the qualification of MBA.

$\Pr(A)=0.6, \Pr(B)=0.48, \Pr(B|A)=0.72$

2(b)(i) $\Pr(A\cup B) = \Pr(A)+\Pr(B)-\Pr(A\cap B)$

$= \Pr(A)+\Pr(B)-\Pr(A)\Pr(B|A)$

$= 0.6+0.48-0.6\times0.72 = 0.648$

2(b)(ii) $\Pr\left(B|\bar{A}\right)=\dfrac{\Pr\left(\bar{A}\cap B\right)}{\Pr\left(\bar{A}\right)}=\dfrac{\Pr\left(B\right)-\Pr\left(A\cap B\right)}{1-\Pr\left(A\right)}=\dfrac{\Pr\left(B\right)-\Pr\left(A\right)\Pr\left(B|A\right)}{1-\Pr\left(A\right)}$

$$=\frac{0.48-0.6\times0.72}{1-0.6}=0.12$$

2(b)(iii) $\Pr\left(\bar{A}|\bar{B}\right)=\dfrac{\Pr\left(\bar{A}\cap\bar{B}\right)}{\Pr\left(\bar{B}\right)}=\dfrac{1-\Pr\left(A\cup B\right)}{1-\Pr\left(B\right)}=\dfrac{1-0.648}{1-0.48}=\dfrac{44}{65}$

2(c) Let A be the event of selected staff is satisfied with the working environment

Let B_1, B_2, B_3 be the event of selected staff works in Hong Kong Island, Kowloon and New Territories, respectively.

$\Pr\left(B_1\right)=0.4, \Pr\left(B_2\right)=0.35, \Pr\left(B_3\right)=0.25,$

$\Pr\left(A|B_1\right)=0.8, \Pr\left(A|B_2\right)=0.6, \Pr\left(A|B_3\right)=0.75$

$$\Pr\left(B_1|A\right)=\frac{\Pr\left(B_1\right)\Pr\left(A|B_1\right)}{\Pr\left(B_1\right)\Pr\left(A|B_1\right)+\Pr\left(B_2\right)\Pr\left(A|B_2\right)+\Pr\left(B_3\right)\Pr\left(A|B_3\right)}$$

$$=\frac{0.4\times0.8}{0.4\times0.8+0.35\times0.6+0.25\times0.75}=\frac{0.32}{0.7175}=\frac{128}{287}\approx0.4460$$

3(a)(i) Let X be the daily cash withdrawal amount from this ATM.

$X \sim N(500000,\ 200000^2)$

$P(380,000 < X < 800,000) = P(-0.6 < Z < 1.5)$

$=1-0.2743-0.0668=0.6589$

3(a)(ii) Let m be the minimum amount needed.

$P(X > m) = 0.1$ $\quad\quad P(Z > \dfrac{m-500000}{200000}) = 0.1$

$\dfrac{m-500000}{200000}=1.28$ $\quad\quad m=\$756,000$

3(a)(iii) $P(X > 400,000) = P(Z > -0.5) = 0.6915$

Let Y be the number of days having a cash requirement of at least \$400,000 each out of 100 days. $Y \sim b(100, 0.6915)$

$n=100>30$, $0.1<p<0.9$,

$np=100(0.6915)=69.15>5$, $nq=100(0.3085)=30.85>5$

Normal approximation can be used.

$$P(Y<65)\approx P(Y<64.5)=P(Z<\frac{64.5-69.15}{\sqrt{100(0.6915)(0.3085)}})$$

$$\approx P(Z<-1.01)=0.1562$$

3(b)(i) Let X be the number of visits to museums paid by a tourist of a certain city.

$X \sim \mathbf{Po}(3)$

$$P(X < 3) = e^{-3}\left(\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!}\right) = 0.4232$$

3(b)(ii)
$$P(X < 6 \mid X \ge 3) = \frac{P(3 \le X < 6)}{P(X \ge 3)}$$
$$= \frac{e^{-3}\left(\frac{3^3}{3!} + \frac{3^4}{4!} + \frac{3^5}{5!}\right)}{1 - 0.4232} = 0.8545$$

4(a) Let X be the period of time (days) that an American tourist stays in Country A.

$\because X \sim N(10.55,\ 3.8^2)$ we have $\overline{X} \sim N\left(10.55,\ \frac{3.8^2}{20}\right)$.

$$P(8 \le \overline{X} \le 12) = P(-3.00 \le Z \le 1.71) = 1 - 0.00135 - 0.0436 = 0.95505$$

4(b)(i)
$$\sum x = 19065, \quad \sum x^2 = 51099825,$$
$$\bar{x} = \frac{19065}{8} = 2383.125(\textit{euro dollars})$$
$$s = \sqrt{\frac{51099825 - 19065^2/8}{8-1}} = 899.6465(\textit{euro dollars})$$
$$\nu = 8 - 1 = 7, \quad t_{0.005;7} = 3.499.$$

Therefore, a 99% confidence interval for the mean expenditure of all French tourists is:

$$2383.125 \pm 3.499\frac{899.6465}{\sqrt{8}} = (1270.1873, 3496.0627)(\textit{euro dollars})$$

4(b)(ii)
$$Z_{0.025}\frac{500}{\sqrt{n}} < 100 \Rightarrow 1.96\frac{500}{\sqrt{n}} < 100 \Rightarrow n > 96.04$$

4(c) Let $\hat{p}$ be the sample proportion of the companies that have investments in Country A.

Then we have $\hat{p} = \frac{579}{1194}$. A 98% confidence interval for the population proportion is:

$$\frac{579}{1194} \pm Z_{0.01}\sqrt{\frac{\left(\frac{579}{1194}\right)\left(1 - \frac{579}{1194}\right)}{1194}} = \frac{579}{1194} \pm 2.33\sqrt{\frac{356085}{1194^3}} = (0.4512, 0.5186)$$