# Analysis of Estimable Link Parameter for Accuracy Improvement of 6-axis Serial Robot

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The 6-axis serial robots capable of driving 6 degrees of freedom are widely used as industrial robots because they can be applied to a wide range of applications. In order to apply a 6-axis serial robot in a high-precision process that requires high repeatability and positional accuracy, a calibration process to reduce the difference between the kinematics model and the actual system is required. Since it is difficult and expensive for end-users to make changes to the mechanical structure or design, it is reasonable to improve the accuracy of 6-axis serial robots by calibrating the robot link parameters in the kinematic model. This paper proposes a method for analyzing estimable link parameters among the kinematic parameters of a 6-axis serial robot. The estimable robot parameters were analyzed using the robot's kinematic error model, the radius deviation from circular test of double ball-bar measurement system, and the tool center point (TCP) position error model.

## NOMENCLATURE

$a_i$ = distance from $z_i$ axis to $z_{i+1}$ axis along $x_i$ axis
$\alpha_i$ = angle from $z_i$ axis to $z_{i+1}$ axis about $x_i$ axis
$d_i$ = distance from $x_{i-1}$ axis to $x_i$ axis along $z_i$ axis
$\theta_i$ = angle from $x_{i-1}$ axis to $x_i$ axis about $z_i$ axis
$R$ = radial distance between circular path's center and a measured point.

## 1. Introduction

The 6-axis serial robots capable of driving 6 degrees of freedom are applied to various processes and widely used as industrial robots[1]. In order to apply the 6-axis serial robot to high-precision processes that require high repeatability and accuracy such as precision measurement and processing, compensation is required to reduce the difference between the kinematics model and the actual system. Compensation for the robot requires a step of calculating the value of the error using the generated error model and measurement data, and it is necessary to improve the accuracy of the 6-axis serial robot by compensating for the robot link parameters in the kinematics model.

This paper proposes a method to analyze the estimable link parameters among the kinematics parameters of a 6-axis serial robot.

## 2. Estimable Link Parameters Analysis

### 2.1 Link Parameters Estimation

The method using four DH(Denavit-Hartenberg) parameters, namely $a, \alpha, d, \theta$, is widely used to for kinematic analysis of a serial robot. For a 6-axis serial robot, 24 link parameters are used for design and these parameters should be calibrated and compensated at actual robot by estimate parameters with measurement data.

Fig.1 A schematic of 6-axis serial robot (KR 60HA, KUKA)

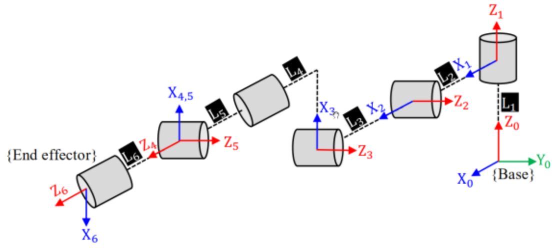


Fig. 2 Kinematics for KR 60HA

Error model to estimate link parameters is derived from robot's kinematic error ($\Delta\mathbf{p}_{tcp}$)[2], radial deviation using double-ball bar ($\Delta\mathbf{R}$), and TCP position error ($\Delta\boldsymbol{\rho}$) in Equation (1), (2), (3).

$$\mathrm{R}\Delta\mathrm{R} = (\mathbf{p}-\mathbf{p}_c)^{\mathrm{T}}(\Delta\mathbf{p}-\Delta\mathbf{p}_c) \quad (1)$$

$$\Delta\mathbf{p}_{tcp} = \boldsymbol{B}\,\Delta\boldsymbol{\rho} \quad (2)$$

$$\Delta\mathbf{R} = \mathbf{U}\Delta\boldsymbol{\rho}\ ,\mathbf{U} = \left(\frac{\mathbf{p}-\mathbf{p}_c}{R}\right)^{T}[\mathbf{B}-\mathbf{B}_c] \quad (3)$$

Where, $\mathbf{p}_c$ is center of the circle, $\mathbf{p}$ is measurement point on the circle, $\mathbf{B}$ is matrix with component of position.

## 2.2 Simulation

Simulations were conducted to theoretically confirm the applicability of kinematic parameters. The circular test path is located on the nominal sphere within the robot's workspace, and the radius is set to 300 mm on the three planes XY, YZ, and ZX as shown in Fig.3.

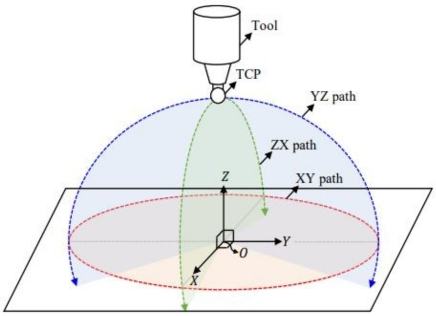


Fig. 3 Generated measurement paths for the circular test

Some of the kinematic errors don't affect the radial deviation at the circular test, which can be confirmed from the fact that the matrix U in Equation (3) is not full-rank. For a rank-deficient matrix, its inverse is less reliable, therefore singular value decomposition is used to make the full-rank matrix as in Equation (4) [3,4,5].

$$\mathbf{U} = \mathbf{M}\boldsymbol{\Sigma}\mathbf{V}^{\mathrm{T}} \quad (4)$$

$$\boldsymbol{\Sigma} = \begin{bmatrix} \sigma_1 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & \sigma_{N-1} & 0 \\ 0 & 0 & 0 & 0 & \sigma_N \end{bmatrix}$$

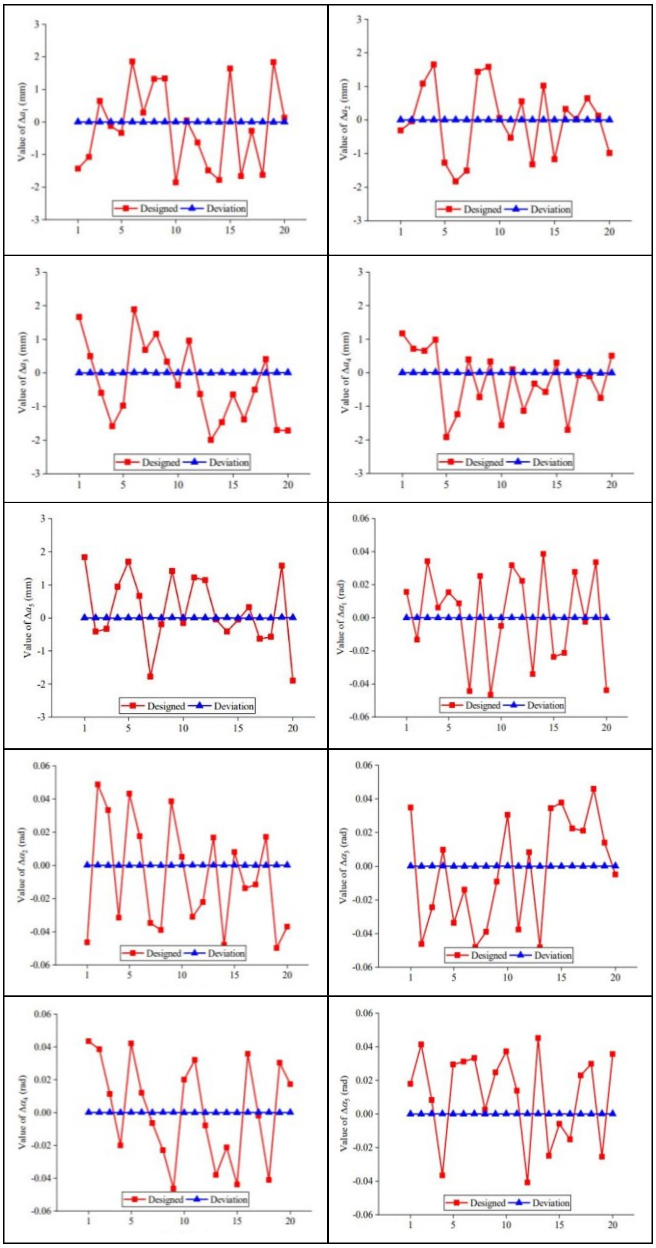
Where, $\mathbf{M}$, $\mathbf{V}$ are orthogonal matrices, and $\boldsymbol{\Sigma}$ is a diagonal matrix consisting of singular value($\sigma$). From the Equation (4), parameters ($\Delta d_1, \Delta d_2, \Delta d_3, \Delta d_6$) that don't affect the output of the system were removed. The offset $\Delta\theta_1 \sim \Delta\theta_6$ of each joint axis was also excluded from the estimation target because $\Delta\theta_1 \sim \Delta\theta_6$ is a position-dependent parameters. As a result, estimable link parameters for KR 60 HA robot is shown in Equation (5).

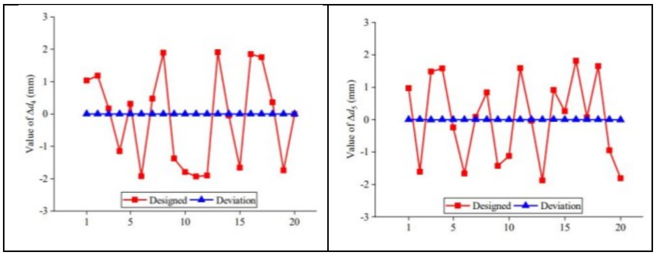
$$\Delta\boldsymbol{\rho}_{estimable} = [\Delta a_1 \cdots \Delta a_5\ \ \Delta\alpha_1 \cdots \Delta\alpha_5\ \ \Delta d_4\ \Delta d_5] \quad (5)$$

For simulation, the radial deviation along the circular path in Fig. 3 is calculated using a kinematic model by applying a randomly generated design errors of ±2mm for length components ($\Delta a_1 \sim \Delta a_6$, $\Delta d_4, \Delta d_5$) and ±0.05rad for angle components ($\Delta\alpha_1 \sim \Delta\alpha_6$). Considering the repeatability of the measuring instrument Ball-bar, random noise within the range of ±1μm is added. Kinematic errors were estimated from the calculated radius deviation and compared with design errors. This process was performed a total of 20 times, and Table 1 shows the deviation between the value of the design error assigned and the estimated error.

Simulation results indicate that the estimated error and design error are the same within a negligible error range. Therefore, the proposed method is suitable for estimating the actual kinematic errors of the robot.

Table 1 Generated values of $\Delta a_{1\sim5}, \Delta\alpha_{1\sim5}, \Delta d_{4\sim5}$ and the deviations with the estimated value





## 3. Conclusions

A mathematical model between radial deviation, TCP position error, and kinematic error and an estimation method are suggested to analyze the link parameters and find the estimable parameters of the 6-axis serial robot. This model and method are applied to the 6-axis serial robot, and estimable parameters among 24 link parameters are determined through singular value decomposition. Simulation results using estimable parameters show that the estimated errors and design errors assigned are the same within a negligible range and the proposed method is suitable for estimating the actual 6-axis serial robot.

## ACKNOWLEDGEMENT

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